Chapter 7:

note: if more than one object just add $M_c x_c \dots / M_c \dots$

①
$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$

Where m is mass & x is distance from the origin.

note : if mass is unknown then use $m = \rho \cdot V$

Chapter 8:

①
$$\tau = r \cdot F \cdot \sin\theta$$

where r is the distance from the axis of rotation and θ is the angle between the force and the lever.

note: clockwise torque is negative & anticlockwise torque is positive.

Unit: N.m

Chapter 9:

① Object at equilibrium:

$$\sum \text{Force} = 0$$

$$\sum \text{Torque} = 0$$

note: pick axis of rotation so that one of the components zeros.

note: dont forget normal force ON the board / object in equilibrium

note: to know if clockwise or anti-clockwise. Hold a pen with one hand at a pivot point and apply force in direction of force with other hand

②

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

Unit: N/m^2

③

$$\text{Strain} = \frac{\Delta l}{l}$$

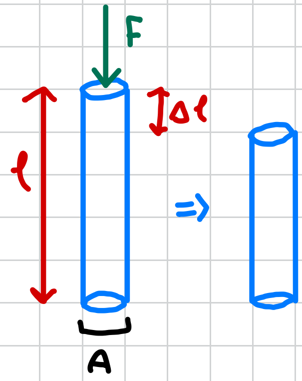
note: Always take Δl as positive even if compression

where Δl is change in length

④ Young Modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l / l}$$

Unit: N/m^2



note: E measures how much a material can resist a change in length when force is applied. E = stress / strain

More stress for same Δl (strain) the more E. So more E can withstand more force.

Likewise, more Δl (strain) for same stress then the less E.

⑤ Bulk Modulus, B:

$$B = \frac{\text{Stress}}{\text{strain}} = \frac{F/A}{\Delta V/V}$$

Unit: N/m^2

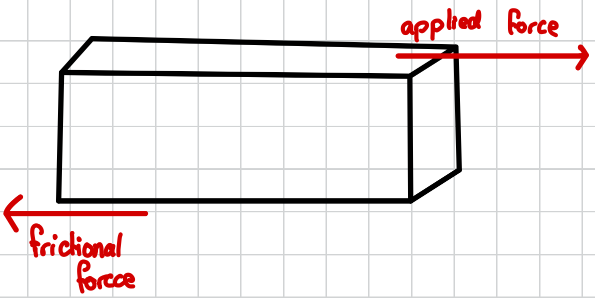
where ΔV is change in volume, even if compression make ΔV positive. A is total surface area. For B we need total S.A not cross sectional area.

Bulk modulus follows same trends as young modulus, so solids have a higher B than liquids & gases because solids need more force (stress) for the same ΔG (strain)

⑥ Shear Modulus, G:

$$G = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta \ell / h}$$

where $\Delta \ell$ is the amount that the object got slanted by and h is the height of the object.



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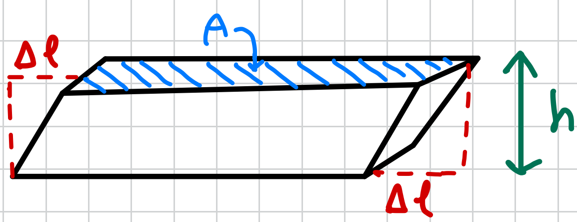


⑦ Hookes law:

$$F = -kx$$

F: applied force N

k: a constant N/m

x: change in length. m

F is -ve when there is increase in length

F is +ve when there is decrease in length.

⑧ Ultimate Strength:

Is the maximum stress an object can withstand before breaking. Measured in N/m^2

A safety factor of x means that the stress applied to an object should not exceed $\frac{1}{x} \times$ ultimate strength of that object.

For example, wood (Tensile ultimate strength of 40×10^6 N/m^2), given a saftey factor of 5, we shouldnt increase the tensile stress on wood by more than $\frac{1}{5} \times 40 \times 10^6$

Chapter 10:

① Density, ρ:

$$\rho = \frac{\text{mass}}{\text{volume}}$$

units: g/cm^3 OR kg/m^3] $1000\ kg/m^3 \Leftrightarrow 1\ g/cm^3$

② Specific Gravity, SG:

$$SG = \frac{\text{density of substance}}{\text{density of } 4^\circ C \text{ water}}$$

$\Rightarrow \rho_{water} = 1\ g/cm^3 = 1000\ kg/m^3$

③ Pressure:

$$P = \frac{\text{Force}}{\text{Area}}$$

Units: N/m^2 = Pascal (P)

$$P = \rho_f \cdot g \cdot h$$

$\Rightarrow$ equation for pressure due to fluid at any point at a certain depth in a fluid.

$$\Delta P = \rho_f \cdot g \cdot (h_2 - h_1)$$

(atmospheric pressure cancels out)

The pressures at any two points in the same depth are the same.

④ Gauge and atmospheric pressure:

$$P_{absolute} = P_{guage} + P_{atmosphere}$$

⑤ Pascal's Principle:

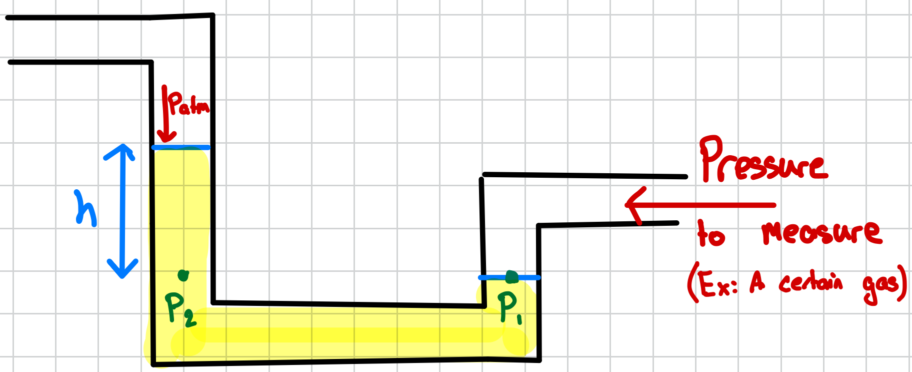
The pressure at two points at the same depth are equal.

At the same point: $$P_{in} = P_{out} \Rightarrow \frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

For the same pressure, $P = \frac{F\uparrow}{A\uparrow}$, if A_{out} increases then F_{out} increases. This is used in hydraulic press, breaks etc.

note that $WORK_{in} = WORK_{out} \Rightarrow \Delta H_{in} > \Delta H_{out}$

⑥ Open-Tube Monometer:



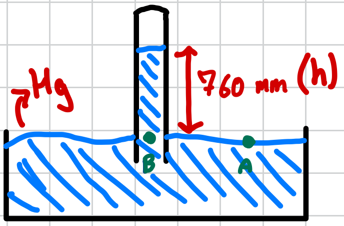
$$P_1 = P_2$$

$$P_{gas} = P_f + P_{atm}$$

$$P_{gas} = \rho_f \cdot g \cdot h + P_{atm}$$

$P_{atm} \rightarrow 101.3 \times 10^3\ Pa$

⑦ Mercury Barometer:



$$P_A = P_B$$
$$P_{atm} = \rho_f \cdot g \cdot h$$

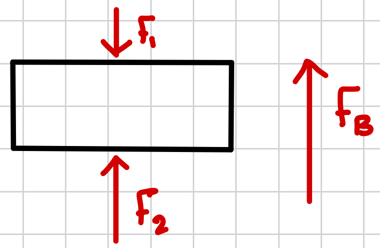
$\therefore \; h = \dfrac{P_{atm}}{\rho_f \cdot g}$

Same P. $\rho \uparrow \Rightarrow h \downarrow$

⑧ Buoyancy and Archimedes Principle:

$$F_B = \rho_f \times g \times V_{sub}$$
$$F_B = m_F \, g$$
↳ weight of displaced fluid

] fully submerged



$F_B = F_2 - F_1$

$$\rho_f \cdot V_{sub} = \rho_o \cdot V_o$$

(partially submerged)

V_{sub} = volume of object under the fluid.

V_o = volume of entire object

⑨ Flow rate and the equation of continuity:

Assuming laminar flow and non-viscous fluid and incompressible fluid:

↳ pressure doesnt change volume so V constant & m constant so $\rho = \frac{m}{V}$ is constant.

* Volume flow rate: $\frac{\Delta V_1}{\Delta t} = \frac{\Delta V_2}{\Delta t}$ (same)

$\therefore \frac{\Delta V}{\Delta t}$ m^3/s

* Mass flow rate: $\frac{\Delta m}{\Delta t} = \frac{\rho \cdot \Delta V}{\Delta t}$

$\therefore \frac{\Delta m}{\Delta t}$

* Continuity Equation:

$$\rho_1 A_1 \frac{\Delta l_1}{\Delta t} = \rho_2 A_2 \frac{\Delta l_2}{\Delta t} \Rightarrow \rho_1 A_1 \cdot v_1 = \rho_2 \cdot A_2 \cdot v_2 \xrightarrow[\text{constant } \rho]{\text{incompressable}} A_1 v_1 = A_2 v_2$$

$$\therefore \quad A_1 v_1 = A_2 v_2$$ ☆

Volume flow rate $= \frac{\Delta V}{\Delta t} = A_1 \times V_1 = A_2 V_2$

Mass flow rate $= \rho \times A_1 V_1$

$A \propto \frac{1}{v}$. Less area the faster the velocity of the fluid.

$AV =$ constant

⑩ Bernoulli's Equation :

How to memorize: $P_1 + k.E_1 + P.E_1 = P_2 + k.E_2 + P.E_2$ but replace m with ρ.

Assuming laminar flow and non-viscous fluid and incompressible fluid :

☆

$$P_1 + \frac{1}{2} \times \rho \times v_1^2 + \rho . g . h = P_2 + \frac{1}{2} \times \rho \times v_2^2 + \rho \times g \times h$$

$\rho . g . h = \frac{1}{V} \times m . g . h \Rightarrow$ P.E per unit volume

$\frac{1}{2} . \rho . v^2 = \frac{1}{2} . m . v^2 . \frac{1}{V} \Rightarrow$ k.E per unit volume

note: $P_1 + \frac{1}{2} \times \rho \times v_1^2 = \downarrow P_2 + \frac{1}{2} \times \rho \times v_2^2 \uparrow$ $\quad P \propto \frac{1}{v}$

As speed increases → pressure decreases. We can see this in everyday life like vacuum, the motor inside spins and air moves fast so inside the vacuum is high speed → low pressure air and outside is low speed high pressure air so air and dust goes from out ($P\uparrow$) to in ($P\downarrow$).

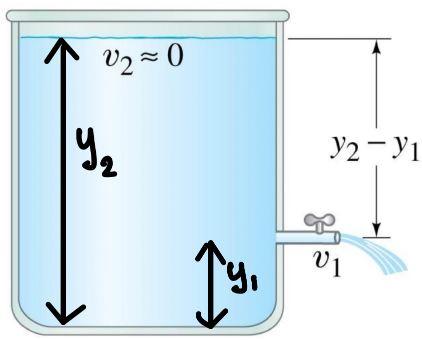
The pressure difference can generate so much force that it moves whole airplanes! Above the wing is high speed low pressure air and below the wing is low speed high pressure air, this difference in pressure generates so much force that it lifts an entire airplane.

(11) Torricelli's Theorum:

Measures speed of fluid coming out from a hole in a tank.

$$v = \sqrt{2g\,(h_2 - h_1)}$$ ☆

where $h_2 - h_1$ is the height from the top of the water till the hole.

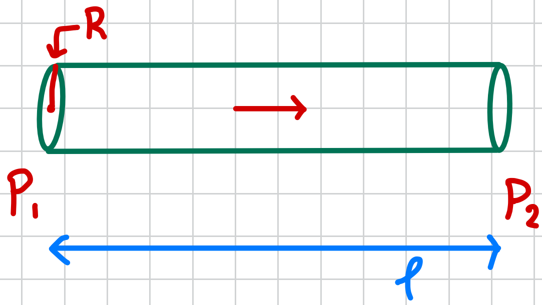


(12) Poiseuille's Equation:

non-compressable non-viscous fluid: Continuity equation and bernoullis principle.

Viscous fluid non-compressable: Poiseuilles equation

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta \ell}$$ ☆



Q: volume flow rate m^3/s

R: radius m

$P_1 - P_2$: pressure difference Pa

η: coefficient of viscosity $Pa \cdot s$

ℓ: length of tube m

$P_1 > P_2$ in order for viscous fluid to move

* $Q \propto R^4$
  - $2R \rightarrow 16Q$
  - $R/2 \rightarrow Q/16$
* $Q \propto (P_1 - P_2)$

Ch. 23:

① Index of refraction:

$$n = \frac{c}{v}$$

n: index of refraction
c: speed of light in vacuum
v: speed of light in median

$c > v \quad \therefore \quad n \geq 1$ $\qquad n \downarrow \quad v \uparrow$

② Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

n: index of refraction
θ_1: Angle of incidence
θ_2: Angle of refraction

Going from $v\uparrow$ $(n\downarrow)$ to $v\downarrow$ $(n\uparrow)$
↓ θ: incidence angle
↓ α: Refracted angle

* $\theta > \alpha$

* $\underbrace{n_1 \sin\theta_1}_{\text{constant}} = \uparrow n_2 \cdot \sin\theta_2 \downarrow$

so when $n_2 \uparrow$ $\theta_2 \downarrow$
so it refracts more,
goes close to the normal

③ Total Internal Reflection :

$$\sin \theta_c = \frac{n_2}{n_1}$$

$\therefore n_2 < n_1$

θ_c: critical angle

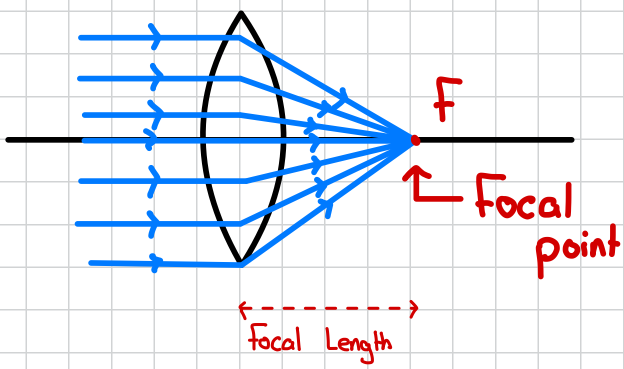
It is the angle of the incident ray that gives a refracted angle of 90°.

Uses: * Binoculars

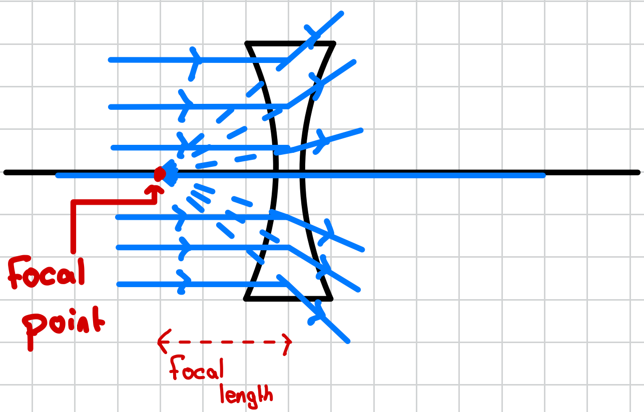
* Fibre optic cables.
  - ↳ WiFi / communication
  - ↳ medicine → images of human organs

Concave Vs Convex lenses :

Convex : converging lens ⇒ memo: convex → converging



Concave : Diverging lens



④ Lens power:

$$P = \frac{1}{f}$$

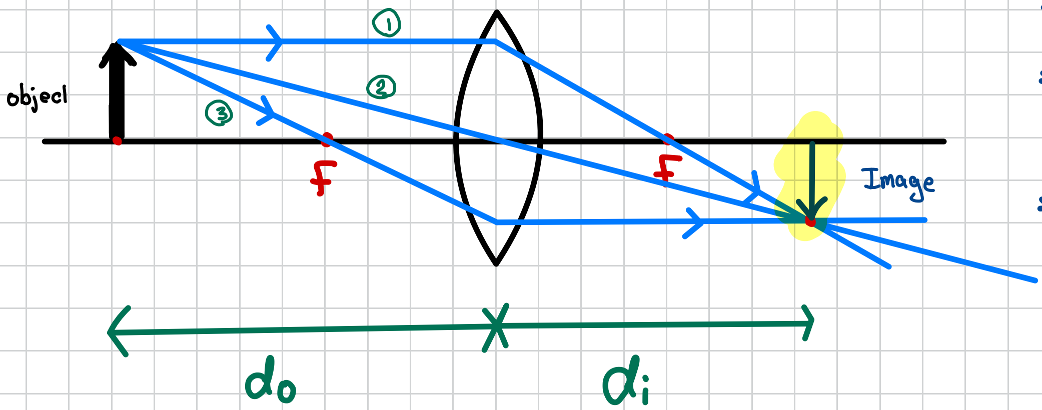
P: Lens's power

f: Focal length

Units: $\frac{1}{m} = m^{-1} = D$ (diopters)

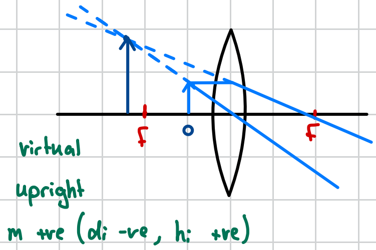
Determining Image using Ray Tracing:

① Convex lens:

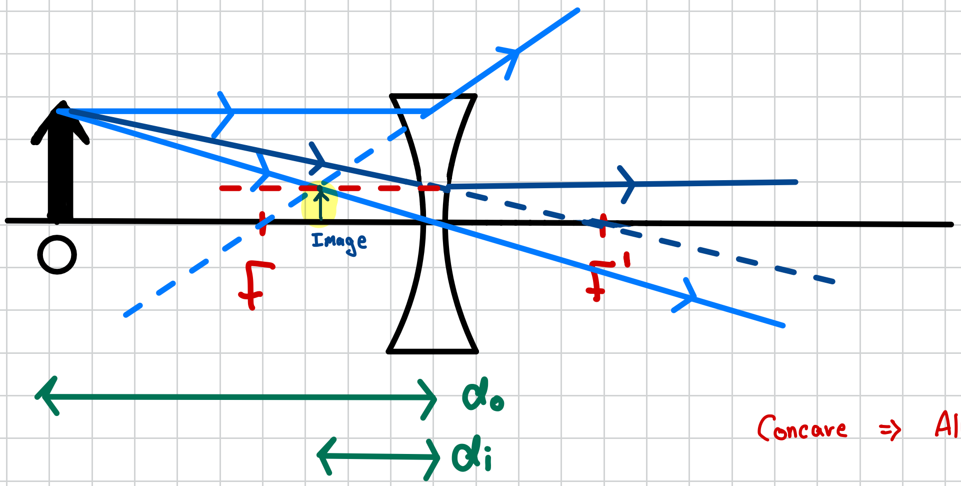


Cuz in opposite side (Cuz result of intersection of rays)

* Image is real
* Image is inverted
* Image can be virtual and upright if object lies below f.



② Concave Lens:



* Image is diminished
* Image is virtual
* Image is upright

Concave => Always virtual & upright

Convex => either real inverted or virtual upright

⑤ Thin Lens Equation:

$$* \; m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$* \; \frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

m: magnification

h_i: height of image

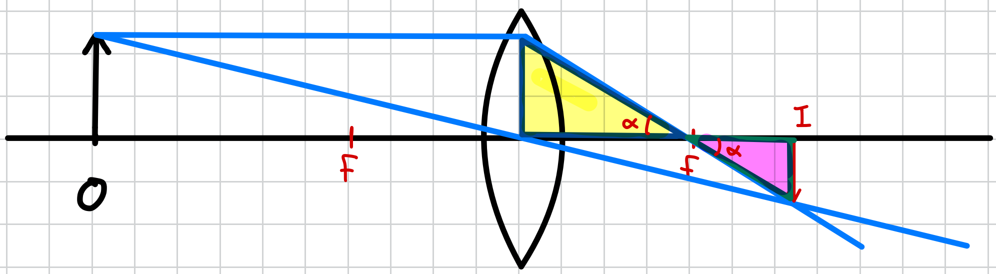
h_o: height of object

d_i: distance of image from center of the lens.

d_o: distance of the object from the center of the lens.

f: focal length

note: Look for similar triangles:



∴ Shaded triangles are equal

$$\frac{Side_a}{Side_d} = \frac{Side_b}{Side_e} = \frac{Side_c}{Side_f}$$

If object too far away from lens:

$d_o = \infty$, $\frac{1}{d_o} = 0$] $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \Rightarrow \frac{1}{d_i} = \frac{1}{f} \Rightarrow$ $\boxed{d_i = f}$

* $|m| > 1 \Rightarrow$ enlarged
* $|m| < 1 \Rightarrow$ reduced
* $|m| = 0 \Rightarrow$ same size

Very important sign conventions:

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

* h_i up → upright image → h_i positive
* h_i down → inverted image → h_i negative
* f/d_i to the right → f/d_i positive
* f/d_i to the left → f/d_i negative

∴ real image ⇒ inverted ⇒ h_i -ve ⇒ d_i +ve ⇒ m -ve
↳ remember convex

∴ virtual image ⇒ upright ⇒ h_i +ve ⇒ d_i -ve ⇒ m +ve
↳ remember concave

Chapter 30:

① Nuclear radius:

$$r = 1.2 \times 10^{-15} \times A^{1/3}$$

r: nucleus radius

A: atomic mass (nucleon number)

units: meters ~ m

Types of radiation:

* α-radiation. Helium nuclei => $_2^4He^{++}$ (basically just nucleus of He cuz $_2^4He$ atom → $_2^4He^{++}$ nucleus)

It has most ionizing power but least penetrating (sheet of paper)

* $\beta^{\pm}$ radiation. β^- is an electron, e^-. β^+ is a positron, e^+.

Can penetrate 3 mm of aluminum.

* Gamma radiation, γ. Is an electromagnetic form of energy. Have zero mass. Large penetrating power

② Radioactive decay law:

$$N = N_0 \, e^{-\lambda t}$$

N: remaining number of radioactive nuclei

N_0: original number of radioactive nuclei

λ: decay constant $\left(\frac{1}{time}\right)$

③ Half life and rate of decay:

$$t_{1/2} = \frac{\ln 2}{\lambda} \quad \therefore \quad t_{1/2} = \frac{0.693}{\lambda}$$

* $t_{1/2} \propto \frac{1}{\lambda}$. $\therefore$ when λ is large the $t_{1/2}$ will be small and thus fast decay

④ Activity (A or R), number of decays per second:

$$A = A_0 \cdot e^{-\lambda t}$$

$$A = \lambda N$$

A: activity / rate of decay

A_0: initial / original activity

N: number of radioactive nuclei

λ: decay constant

Imp: Units is $\frac{decay}{second}$ (also s^{-1} works) so make sure you convert years to seconds.

S.I: 1 Bq = 1 decay/s OR 1 Ci (curie) = 3.7×10^{10} decay/s

(5) Mean life of an isotope :

$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693}$$

Chapter 31:

Measuring radiation, Dosimetry:

Activity doesn't give information regarding the effect of radiation on biological tissues.

① Absorbed dose:

The energy deposited per kg in any median by any radiation type.

* Grey (Gy): 1 Gy = 1 J/kg

* Rad: 1 Rad = 0.01 J/kg

⇒ **1 Gy = 100 Rad**

Absorbed dose = $\frac{\text{energy}}{\text{mass}}$ ∴ $AD = \frac{E}{m}$

make sure to always convert Rad to Gy, using 1 Gy → 100 Rad

A.D doesn't show how dangerous a type of radiation is, see effective dose.

$$E_{tot} = A \times E_\gamma$$

E_{tot} ↓ Total energy J/s

$A \Rightarrow \frac{\text{decay}}{\text{second}}$

$E_\gamma \Rightarrow \frac{\text{Energy}}{\text{decay}}$

$\Rightarrow \frac{\text{dec}}{\text{sec}} \times \frac{E}{\text{dec}} \Rightarrow \frac{E}{s} = \frac{J}{s}$

② Effective dose :

$$E.D = A.D \times R.B.E$$

R.B.E : relative biological effectiveness. => Higher R.B.E , the more dangerous the radiation

A.D : absorbed dose

∴ $E.D = A.D \times R.B.E \uparrow$
↓ more dangerous

E.D : effective dose

Units:

	A.D		E.D
	Grey (Gy)	⟶	Sievert (Sv)
	Rad	⟶	rem

∴ 1 Gy = 100 Rad
1 Sv = 100 rem

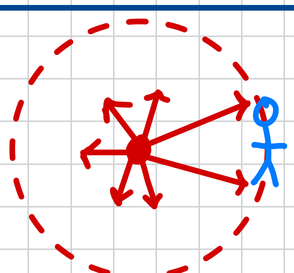
③ Radiation Intensity:

$$I \propto \frac{1}{r^2}$$

] smaller radius then more intensity. (closer to source)

$$* \ \frac{\text{Energy intercepted by body}}{\text{Total radiated energy}} = \frac{S.A/A \text{ of body}}{4 \pi r^2}$$

↳ S.A of sphere



* $N = \frac{E_{tot}}{E_\gamma}$

$\Rightarrow A = N\lambda$

* $E_{tot} = A \times E_\gamma$

$E_{tot} \rightarrow \frac{J}{s}$

$A \rightarrow \frac{Dec}{s} \times \frac{E\,(J)}{Dec}$