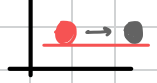
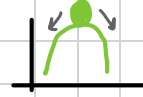


Chapter 9

Lecture 11.2
sections 9.4 - 9.5 - 9.6

Stability and Balance



- **stable equilibrium**: Object goes back to its original position.
- **unstable equilibrium**: Object moves further from its original position.
- **neutral equilibrium**: Object remains at its new position.

Elasticity

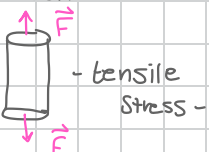
→ Applying External Force → **deformation - change in size, shape**

- Small = After removing \vec{F}
 - it return to its original shape, size.
- past the **Elastic Limit**
 - object remains permanently deformed
- way too beyond **Elastic Limit**.
 - Break, fracture.

→ In Solids : **3 types of deformation.**

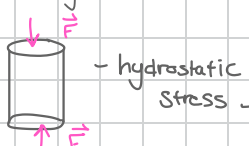
Stretching

"tension"



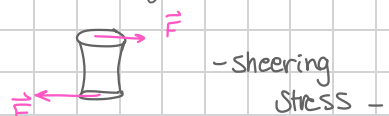
Compression

"pushing"



Shearing

"twisting"



1. Changing The Length of a Solid

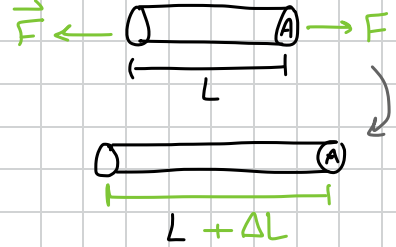


* Force is Applied to the End of an Object.

- Stretching

$$\frac{\vec{F}}{A} = E \frac{\Delta L}{L} \Rightarrow \Delta P = E \frac{\Delta L}{L}$$

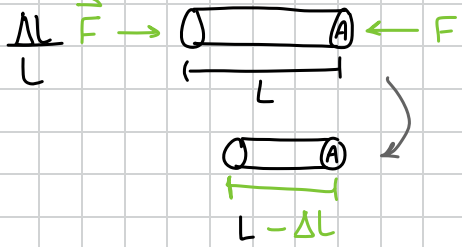
Stress
Young's Modulus
Strain



- Compression

$$\frac{\vec{E}}{A} = -E \frac{\Delta L}{L} \Rightarrow \Delta P = -E \frac{\Delta L}{L}$$

Stress
Young's Modulus
Strain



$$\Delta L = \frac{F L}{A E} \rightarrow \text{directly proportional } \Delta L \propto F, L$$

$$\Delta L = \frac{F L}{A E} \rightarrow \text{inversely proportional } \Delta L \propto \frac{1}{E}, \frac{1}{A}$$

- E** young's Modulus
- unit $\cdot \frac{N}{m^2}$ → where $\frac{\Delta L}{L}$ Dimensionless.
 - material property → doesn't depend on size or shape
 - E values are large + slightly different for compression and stretching for some materials, like human bones

$$E_{\text{stretching}} > E_{\text{compression}} \rightsquigarrow \Delta L_{\text{stretching}} < \Delta L_{\text{compression}}$$

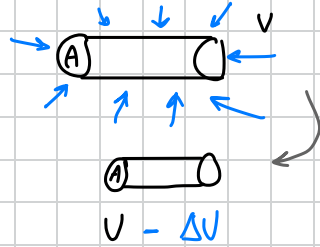
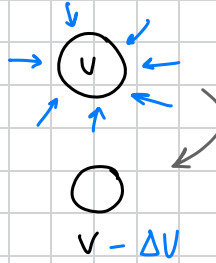
2. Changing the Volume of Solids



* Force is Applied to the Entire Surface Area of an object

$$\frac{F}{A} = -B \frac{\Delta V}{V} \Rightarrow \Delta P = -B \frac{\Delta V}{V}$$

$\frac{F}{A}$ Stress $-B$ Bulk Modulus $\frac{\Delta V}{V}$ Strain



* Volume decreases when pressure increases.

$$\Delta V = - \frac{F V}{A B}$$

ΔV = decrease in volume $\frac{F V}{A B}$

\sim directly proportional \sim inversely proportional.

Bulk Modulus

→ unit: $\frac{N}{m^2}$ → $\frac{\Delta V}{V}$ dimensionless

→ Always (+)

→ Applies to Fluids & Solids → Fluids \Rightarrow Volume

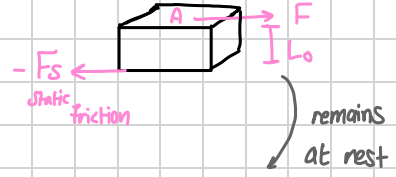
→ $B_{\text{gases}} < B_{\text{liquids}} < B_{\text{solids}}$. → gases are easily compressed

3. Changing the Shape of a solid

* Force Applied to the top surface of an object

$$\frac{F}{A} = G \frac{\Delta x}{L_0} \Rightarrow \Delta P = G \frac{\Delta x}{L_0}$$

$\frac{F}{A}$ Stress
 G Shear Modulus
 $\frac{\Delta x}{L_0}$ Strain
 L_0 thickness



* The object becomes Slanted

by the amount Δx



$$\Delta x = \frac{L_0 F}{G A}$$

L_0 directly proportional
 F directly proportional
 G inversely proportional
 A inversely proportional

• G Shear Modulus

→ unit : $\frac{N}{m^2}$ → $\frac{\Delta x}{L_0}$ dimensionless

→ Always +

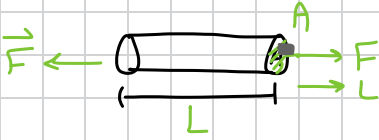
→ G values are large → large Force causes small Shear

→ $G < E$ For All solids $\leadsto G = \frac{1}{3} / \frac{1}{2} E$

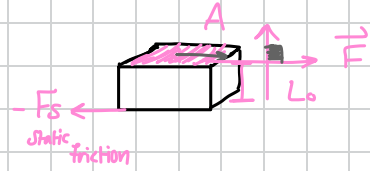
Except the human bones $\leadsto G_{HB} > E_{HB}$ *resistance for fracture in shear is 5 times greater than in tension.

• Comparing Eq. 1 & Eq. 3

$$\frac{F}{A} = E \frac{\Delta L}{L}$$



$$\frac{F}{A} = G \frac{\Delta x}{L_0}$$



• $L \Rightarrow$ Length

\rightarrow measured in the direction of \vec{F}

• $A \Rightarrow$ cross sectional Area

\rightarrow is perpendicular to \vec{F}

• $L_0 \Rightarrow$ thickness

\rightarrow measured in a direction perpendicular to \vec{F} .

• $A \Rightarrow$ Area of the Surface

\rightarrow is parallel to \vec{F}

• Fracture

\rightarrow up to the proportional limit

if the stress is removed the material returns to its original length

deformation - responds vanishes.

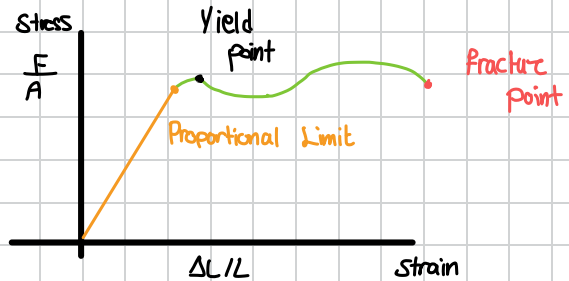
- reversible deformation \rightarrow elastic deformation

\rightarrow Yield point : stress causes sudden deformation without an increase in Force

\rightarrow from Proportional Limit to Fracture point

if the stress is removed the material will not return to its original length

Permanently deformed and it enters Plastic region



→ Additional stress until the Fracture point

• Fracture Point: the material breaks or tears apart.

• Ultimate (breaking) stress: where the maximum elongation is reached

• Ultimate strength = tensile strength = maximum value the material can withstand without breaking.

* it is necessary to maintain 3 to 10 safety factor.

⇒ stresses should not exceed $\frac{1}{10}$ to $\frac{1}{3}$ of the ultimate strengths.