One-Way ANOVA Table-Missing Values

Solved Problems

Problem (1)

Find the missing values (A, B, C, D, E and K) of the partially completed one-way ANOVA table given below if $\alpha = 0.05$:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	2.124	C	0.708	4.5	K
Within	A	20	E		
Total	B	D			

Solution

(1) Between MS = $\frac{\text{Between SS}}{\text{k} - 1} = \frac{\text{Between SS}}{\text{C}}$

Then $\text{C} = \text{k} - 1 = \frac{\text{Between SS}}{\text{Between MS}} = \frac{2.124}{0.708} = 3 \Rightarrow \text{C} = 3$

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(2) $\text{D} = \text{n} - 1 = \text{k} - 1 + 20 = \text{C} + 20 = 3 + 20 = 23 \Rightarrow \text{D} = 23$

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(3) $\text{F} - \text{value} = \frac{\text{Between MS}}{\text{Within MS}} = \frac{\text{Between MS}}{\text{E}}$

Then $\text{E} = \frac{\text{Between MS}}{\text{F}-\text{value}} = \frac{0.708}{4.5} = 0.157 \Rightarrow \text{E} = \text{Within MS} = 0.157$

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(4) $\text{Within MS} = \frac{\text{Within SS}}{\text{n} - \text{k}}$ then $\text{E} = \frac{\text{Within SS}}{\text{n} - \text{k}} = \frac{\text{A}}{\text{n} - \text{k}}$ Thus

$\text{A} = \text{Within MS} = (\text{n} - \text{k}) * \text{E} = 20 * 0.157 = 3.14 \Rightarrow \text{A} = 3.14$

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(5) $\text{Total SS} = \text{Between SS} + \text{Within SS}$ then $\text{B} = \text{Between SS} + \text{A}$

Thus $\text{B} = 2.124 + 3.14 = 5.264 \Rightarrow \text{B} = 5.264$

(6) The approximate $p-value$ (given by the area to the right of F under an $F_{(k-1,\ n-k,\ 1-\alpha)}$ distribution) can be calculated as follows:

$$\begin{aligned} \mathrm{K} = p-value &= P\left(F_{(k-1,\ n-k,\ 1-\alpha)} > F\right) \\ &= 1 - P\left(F_{(3,\ 20,\ 0.95)} \le 4.5\right) \\ &= 1 - 0.99 \\ &= 0.01 < \alpha = 0.05 \ \Rightarrow\ \mathrm{K} = 0.01 \end{aligned}$$

TABLE 8 Percentage points of the *F* distribution ($F_{d_1,d_2,p}$) *(continued)*

df for denominator, d_2	p	*df* for numerator, d_1: 1	2	3	4	5	6	7	8	12	24	∞
20	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.89	1.77	1.61
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.68	2.41	2.09
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
	.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.68	3.22	2.69
	.999	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	4.82	4.15	3.38

Note that

The exact p-value by using Minitab is given as follows:

Cumulative Distribution Function

F distribution with 3 DF in numerator and 20 DF in denominator

x	P(X ≤ x)
4.5	0.985627

$$\mathrm{p}-\mathrm{value} = 1 - 0.985627 = 0.014373 \approx 0.01$$

Therefore, the complete list of the missing values will be given as follows:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	2.124	C = 3	0.708	4.5	K = 0.01
Within	A = 3.14	20	E = 0.157		
Total	B = 5.264	D = 23			

Exercise (1)

Find the missing values (A, B, C, D, E and K) of the partially completed one-way ANOVA table given below if $\alpha = 0.05$:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	28	C	14	7	K
Within	A	15	E		
Total	B	D			

Answer

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	28	$C = 2$	14	7	$K = 0.007$
Within	$A = 30$	15	$E = 2$		
Total	$B = 58$	$D = 17$			

Problem (2)

Find the missing values (A, B, C, D, E, F, K and H) of the partially completed one-way ANOVA table given below if $k = 3, n = 47$ and $\alpha = 0.05$:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	1200	B	E	F	H
Within	A	C	K		
Total	1800	D			

Solution

(1) Total SS = Between SS + Within SS then $1800 = 1200 + A$

Thus $A = 1800 - 1200 = 600$ ➡ $A = 600$

(2) $B = k - 1 = 3 - 1 = 2$ ➡ $B = 2$

(3) $C = n - k = 47 - 3 = 44$ ➡ $C = 44$

(4) $D = n - 1 = 47 - 1 = 46$ OR $D = B + C = 2 + 44 = 46 \Rightarrow D = 46$

(5) $E = \text{Between MS} = \frac{\text{Between SS}}{k-1} = \frac{\text{Between SS}}{B} = \frac{1200}{2} = 600 \Rightarrow E = 600$

(6) $K = \text{Within MS} = \frac{\text{Within SS}}{n-k} = \frac{\text{Within SS}}{C} = \frac{A}{C} = \frac{600}{44} = 13.64 \Rightarrow K = 13.64$

(7) $F = F - \text{value} = \frac{\text{Between MS}}{\text{Within MS}} = \frac{E}{K} = \frac{600}{13.64} = 43.988 \Rightarrow F = 43.988$

(8) The approximate $p - value$ (given by the area to the right of F under an $F_{(k-1,\ n-k,\ 1-\alpha)}$ distribution) can be calculated as follows:

$$H = p - value = P\left(F_{(k-1,\ n-k,\ 1-\alpha)} > F\right)$$
$$= 1 - P\left(F_{(2,\ 44,\ 0.95)} \leq 43.988\right)$$
$$= 1 - 0.9999$$
$$= 0.0001 \approx 0 < \alpha = 0.05 \Rightarrow H = 0$$

Note that

The exact p-value by using Minitab is given as follows:

Cumulative Distribution Function

F distribution with 2 DF in numerator and 44 DF in denominator

x	P(X ≤ x)
43.988	1.00000

$p - \text{value} = 1 - 1 = 0$

Therefore, the complete list of the missing values will be given as follows:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	1200	$B = 2$	$E = 600$	$F = 43.988$	$H = 0$
Within	$A = 600$	$C = 44$	$K = 13.64$		
Total	1800	$D = 46$			

Exercise (2)

Find the missing values (A, B, C, D, E, F, K and H) of the partially completed one-way ANOVA table given below if $k = 3, n = 18$ and $\alpha = 0.05$:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	3.1214	B	E	F	H
Within	A	C	K		
Total	4.6384	D			

Answer

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value	p-value
Between	3.1214	$B = 2$	$E = 1.5607$	$F = 15.4372$	$H = 0$
Within	$A = 1.5170$	$C = 15$	$K = 0.1011$		
Total	4.6384	$D = 17$			

Exercises

Exercise (1)

Complete the one-way ANOVA table given below:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value
Between	704	B	234.67	5.40
Within	652	C	D	
Total	A	18		

Exercise (2)

Complete the one-way ANOVA table given below:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value
Between	100	4	C	E
Within	135	45	D	
Total	A	B		

Exercise (3)

Complete the one-way ANOVA table given below:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value
Between	A	2	C	E
Within	668	B	D	
Total	1.146	26		

Exercise (4)

Complete the one-way ANOVA table given below:

One-Way ANOVA Table

Source of Variation	SS	df	MS	F-value
Between	913.425	5	C	E
Within	A	B	D	
Total	5302.425	82		
