Physics 101

Chapter 3:

Vectors and Scalars

Scalars → Magnitude ONLY

e.g → Distance
→ Speed
→ Mass
→ Temperature

Vectors ↳ Has a magnitude (1) and direction (2)

e.g: → Displacement
→ Velocity
→ Acceleration
→ Force
→ weight

Representing Vectors

→ Tip of the arrow represent direction

↳ length of the Straight line represent the magnitude

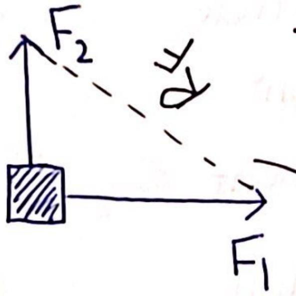
Represent As : A|B|C|D

$|\vec{A}|$ → Represents the magnitude of vectors

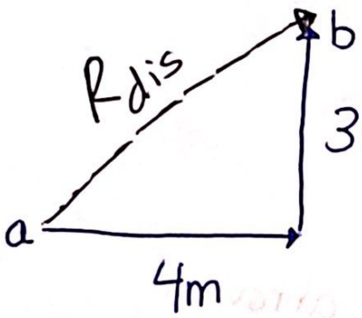
Addition of Vectors

⊛ Adding vectors is different because we must take both magnitude and direction

e.g →

① 

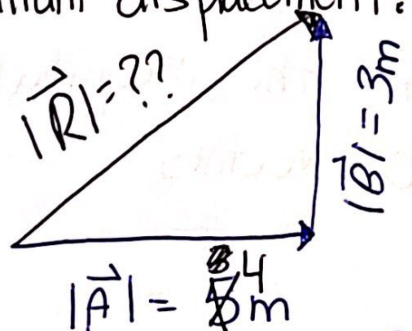
→ Forces can't be simply added, resulting finding Res_F

② 

- There are 2 displacements
- Finding Res_{dis}

Draw both vectors:

Sameeh moved ~~5m~~ 4m along the ~~positax~~ positive x-axis and then ~~3m~~ 3m along the positive y-axis, What is his resultant displacement.



PYTHAGORAS THEOREM

$$R^2 = A^2 + B^2$$

$$R = \sqrt{4^2 + 3^2}$$

~~$R = \sqrt{36} = 6m$~~ $R = \sqrt{25} = 5$

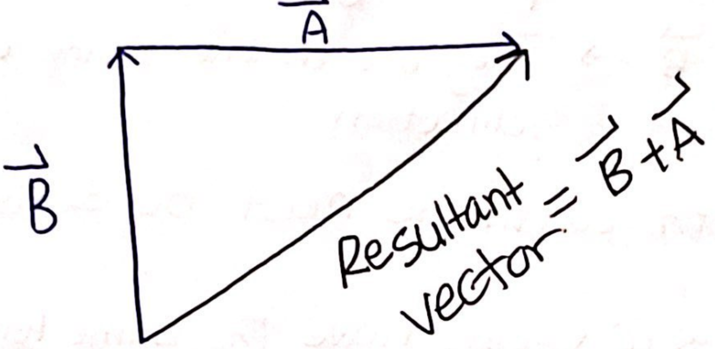
$$|A - B| \leq R \leq \vec{A} + \vec{B}$$

~~$< 5 < 8$~~ $1 < 5 < 7$

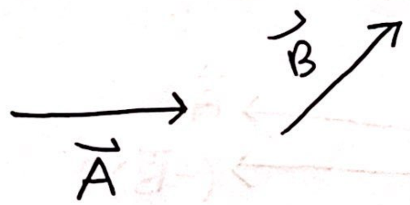
ind $\vec{B}+\vec{A}$

tail of $\vec{A}$ on the tip of $\vec{B}$. The resultant starts from the tail of $\vec{B}$ and ends on the tip of $\vec{A}$

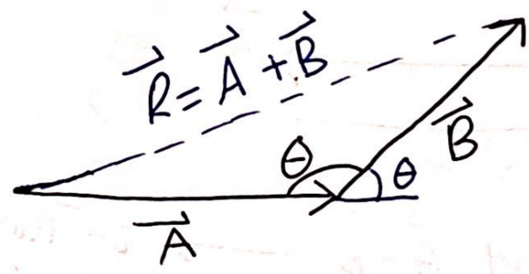
$R = \vec{B} + \vec{A}$



vectors do not form right angled triangles



→ Find $\vec{R} = \vec{A} + \vec{B}$

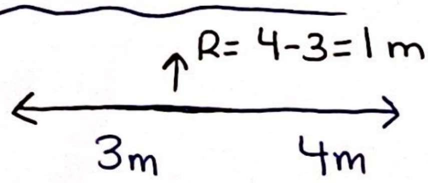


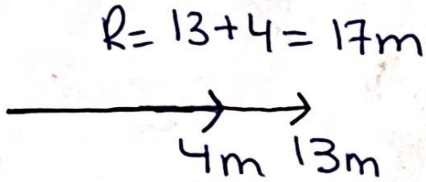
USE THE COSINE RULE ←

θ: when two vectors originate from the same point

$\theta \Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta \Rightarrow R^2 = A^2 + B^2 - 2AB\cos\theta$





Subtraction of vectors

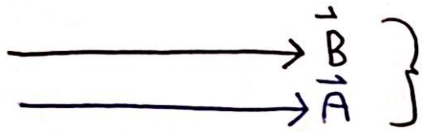
① Equality of Vectors

if $\vec{A} = \vec{B}$

(i) $|\vec{A}| = |\vec{B}|$ → Have equal magnitudes

(ii) $\vec{A}$ is parallel to $\vec{B}$ → They are in the Same direction

NOTE: That both conditions must be Satisfied



① vectors have the Same length
② point to the same direction and Parallel

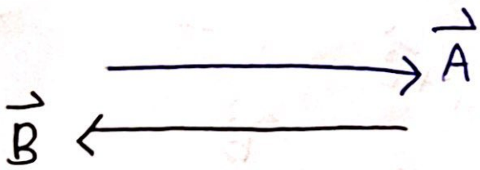
Therefore $|\vec{A}| = |\vec{B}|$

② Negative of a vector

If $\vec{A} = -\vec{B}$

↳ $|\vec{A}| = |\vec{B}|$ → Absolute.

↳ $\vec{A}$ is antiparallel to $\vec{B}$ (Opposite direction)



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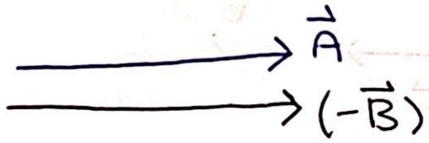
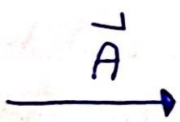
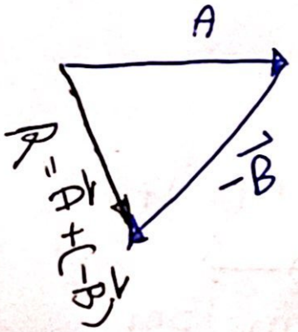


(*) Find $\vec{A} - \vec{B}$ → i- Turn it to addition

$\vec{A} + (-\vec{B})$

Suppose





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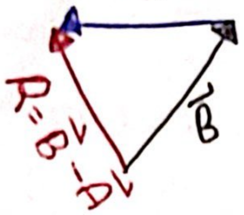
Find $\vec{B}-\vec{A} = \vec{B} + (-\vec{A})$



NOTE : $\vec{A}-\vec{B} \neq \vec{B}-A$

instead $\vec{A}-\vec{B} = -(\vec{B}-\vec{A})$

* $\vec{A}-\vec{B}$ is the negative of $\vec{B}-\vec{A}$
* ~~St~~ Subtraction is NOT Commutative

Multiplication of a vector by a Scalar

Let $|\vec{A}| = 1m$ along the positive x-axis

(i) sketch $\vec{A}$ = $\xrightarrow{\vec{A}}$ 1m

(ii) sketch $\vec{2A}$ = $\xrightarrow{B = 2\vec{A}}$ 2m

this means $|\vec{B}| = 2|\vec{A}|$ are in the same direction and ~~parral~~ parallel.

(iii) $\vec{C} = \frac{1}{2} A$ $\xrightarrow{C = \frac{1}{2}\vec{A}}$ 0.5m

this means the magnitude of C is 1/2 magnitude of A, also they are both in the same direction and parallel.

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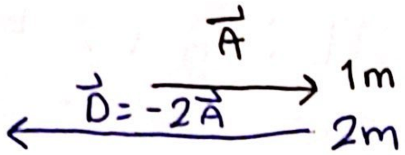
(iv) $\vec{D} = -2\vec{A}$

$\therefore |\vec{D}| = 2|\vec{A}| = 2m$



this means that $\vec{D}$ is antiparallel to $\vec{A}$ (opposite direction)

An example of a vector we usually use, and we multiply it by a number

// Newton's Second law

$$\vec{F} = m\vec{a}$$

force is a vector
Accelaration is a vector
Mass is a Scalar

$$\vec{a} = \frac{1}{m} \cdot \vec{F}$$

mass > 0

$\vec{a}$ ALWAYS parallel to $\vec{F}$

* As the accelaration increases the force increases.

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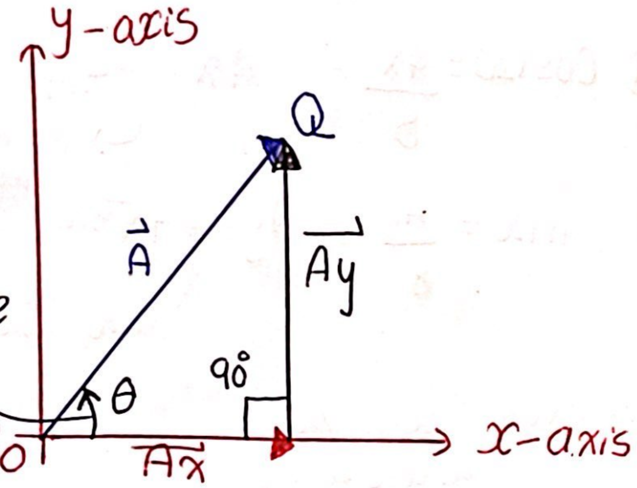
Adding vectors by Components

→ Suppose a Car moved from the Origin to point Q, a displacement $\vec{A}$. Is there an alternative route by moving along the positive x and y axes? The answer is YES



we define our θ
- Respect to x-axis
- Anticlockwise

(i) move displacement $\vec{Ax}$ along the positive x-axis

(ii) turn and make a displacement $\vec{Ay}$ along the positive y-axis

$$\vec{A} = \vec{Ax} + \vec{Ay} \longrightarrow R_{\vec{A}}$$

∴ $\vec{A}$ is the resultant displacement of $\vec{Ax}$ and $\vec{Ay}$

$|\vec{Ax}| \equiv Ax$ is called the x component of $\vec{A}$

$|\vec{Ay}| \equiv Ay$ is called the y Component of $\vec{A}$

$$A^2 = Ax^2 + Ay^2 \quad \text{(Pythagoras theorem)}$$

* SOH CAH TOA

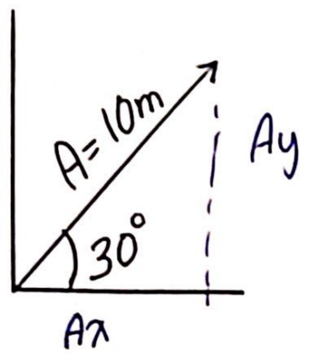
$\cos\theta = \frac{Ax}{A}$ $\quad Ax = A\cos\theta$ — ①

$\sin\theta = \frac{Ay}{A}$ $\quad Ay = A\sin\theta$ — ②

$\tan\theta = \frac{Ay}{Ax} \rightarrow \theta = \tan^{-1}\left(\frac{Ay}{Ax}\right)$

$①^2 + ②^2 \Rightarrow Ax^2 + Ay^2 = A^2(\sin^2\theta + \cos^2\theta) = F$

Examples :



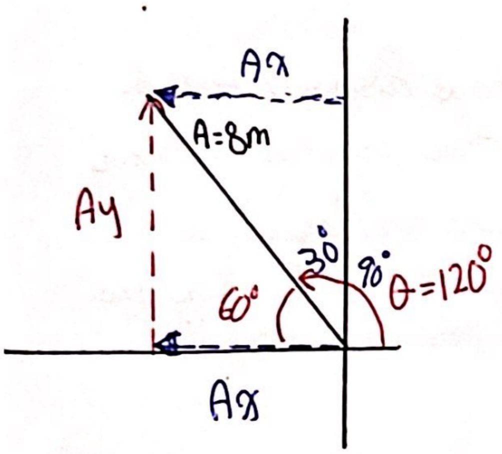
$\frac{Ax}{10} = \cos 30 \rightarrow Ax = 5\sqrt{3}\,m$

↳ $\cos\theta = \frac{Adj.}{hyp.}$

$\frac{Ay}{10} = \sin 30 \rightarrow Ay = 5m$

↓ $\sin\theta = \frac{Opp.}{hyp.}$

Second quadrant	First quadrant
Ay + Ax −	Ay + Ax +
Third quadrant	**Fourth quadrant**
Ay− Ax+	



① $\cos 120 = \frac{Ax}{8} = \quad Ax = -4m$

↳ Along the negative x-axis ←

② $\sin 120 = \frac{Ay}{8} = \quad Ay = +4\sqrt{3}\,m$

↑ along the positive y-direction. ↑

Alternatively : use the acute angle 60°

lies on 2nd quadriant

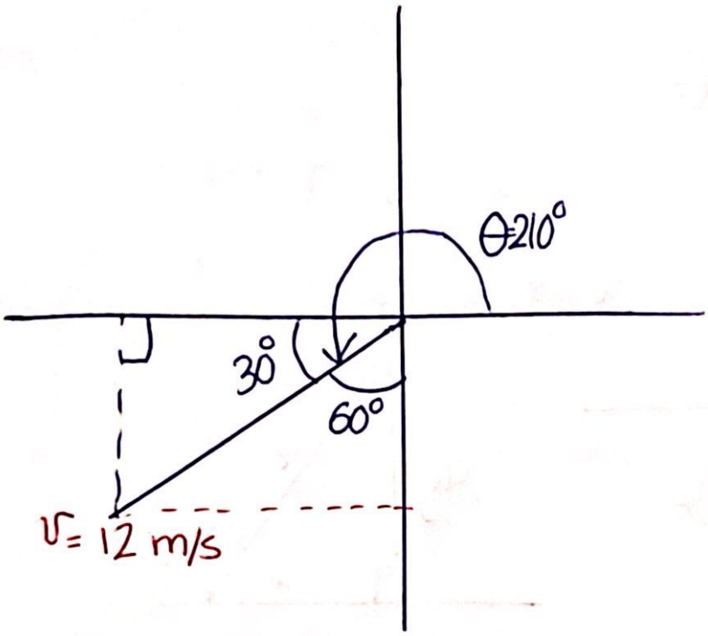
$Ax = -8\cos 60 = -4\ m$

$Ay = 8\sin 60 = 4\sqrt{3}\ m$

NOTE: ⓘ when we use θ with positive x-axis is an anticlockwise direction ⇒ The sign of each component comes out of the angle automatically

ⓘ When we use the acute angle ⇒ we must put the sign of each component by hand depending on the quadrant where the vector is.

xample.



Three different ways to Solve

① Use $\theta = \cancel{120^\circ}\ 210^\circ$ measured with respect to the positive x-axis in an anticlockwise direction

$V_x = 12\cos \cancel{120}\ 210 = -6\sqrt{3}$ m/s
$V_y = 12\sin \cancel{120}\ 210 = -6$ m/s

Both to the negative V_x and V_y direction.
← x-axis, ↓ y-axis

② Using the accute angle

$V_x = -12\cos 30^\circ = -6\sqrt{3}$ m/s (in 3rd quadrant)
$V_y = -12\sin 30^\circ = -6$ m/s (in 3rd quadrant)

③ $V_x = -12\sin 60^\circ = -6\sqrt{3}$ m/s

Look at the graph drawn.

↳ because V_x is opposite to the angle

$V_y = -12\cos 60^\circ = -6$ m/s
↳ because V_y is the adjacent to the angle.

Example :

Find a and θ

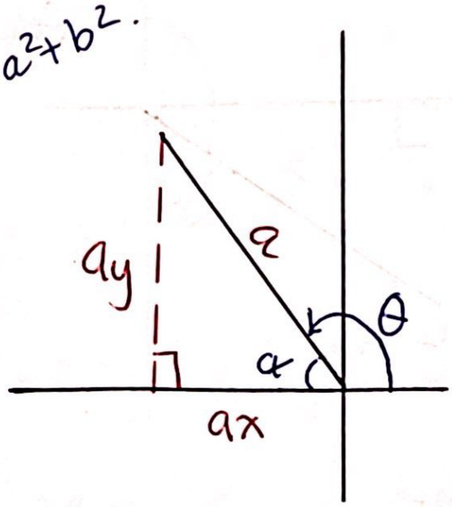
$a_x = -7\sqrt{3}\ m/s^2$, $a_y = 7\ m/s^2$

$a^2 = a_x^2 + a_y^2$ $\quad R^2 = a^2 + b^2$.

Resultant.

$a = \sqrt{(-7\sqrt{3})^2 + 7^2}$

$a = 14\ m/s^2$



$\theta = ??$

$\alpha = \tan^{-1}\left(\frac{a_y}{a_x}\right)$ $\quad \alpha = \tan^{-1}\left(\frac{7}{-7\sqrt{3}}\right)$

$\alpha = 30$

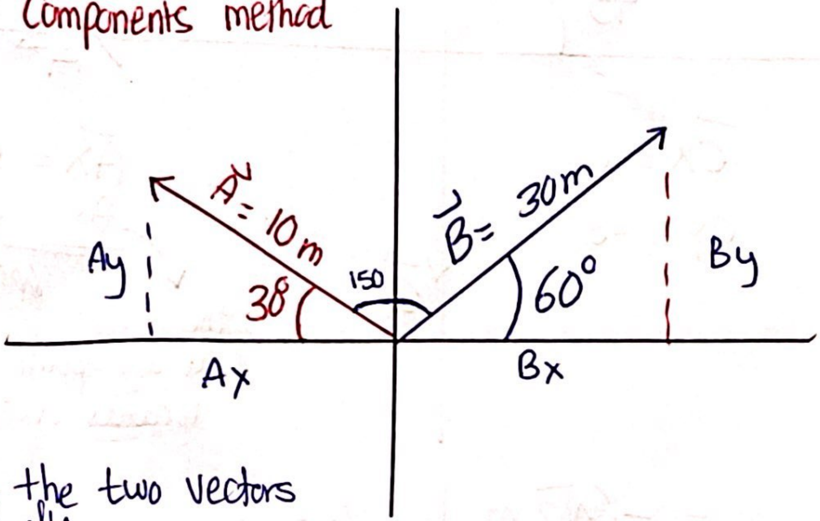
$\theta = 180 - 30 = 150^\circ \longrightarrow$ Second quadrant.

X → - Y → +	X → + Y → +
X → - Y → -	X → + Y → -

Adding Vectors by the Components

Find $\vec{R} = \vec{A} + \vec{B}$

$|\vec{A}| = 10m$ ↳ using the
$|\vec{B}| = 30m$ Components method



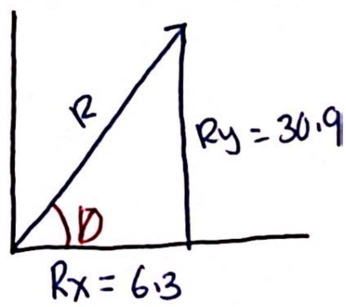
① Solve for the two vectors independantly

$Ax = 10\cos 150^\circ = -5\sqrt{3}\,m$ $Bx = 30\cos 60 = 15m$

$Ay = 10\sin 150^\circ = 5\,m$ $By \quad 30\sin 60 = 15\sqrt{3}\,m$

$Rx = Ax + Bx$

$Rx = -5\sqrt{3} + 15 = \underline{\underline{6.3m}}$



$Ry = Ay + By$

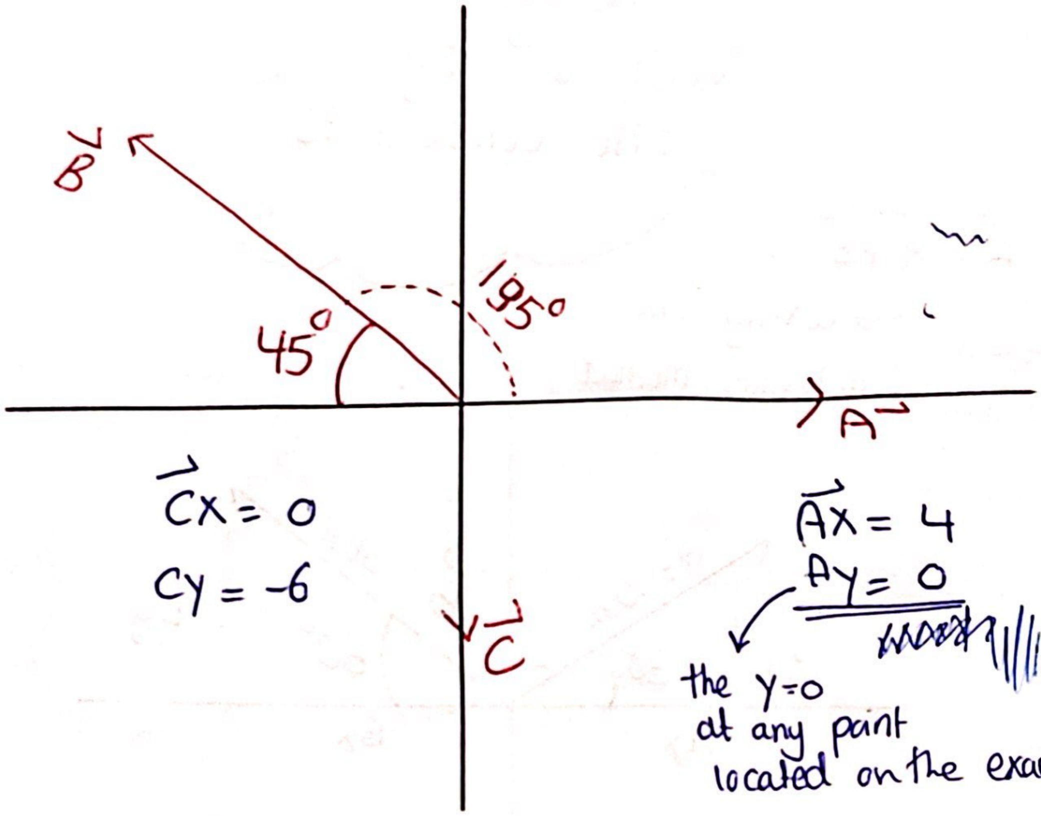
$Ry = 5 + 15\sqrt{3} = \underline{\underline{30.9m}}$

$$R = \sqrt{30.9^2 + 6.3^2}$$

$$R = 31.5m$$

$\theta = \tan^{-1}\left(\frac{30.9}{6.3}\right)$ $\theta = 78.5^\circ$

Example: $|\vec{A}| = 4m$ $|\vec{B}| = 12m$ $|\vec{C}| = 6m$



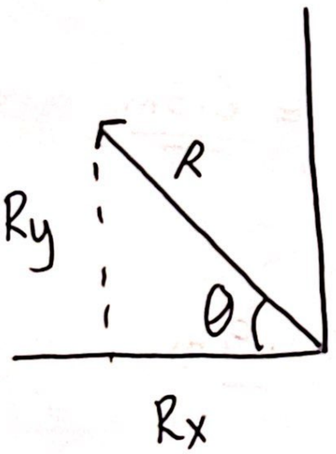
$\vec{C}x = 0$
$Cy = -6$

$\vec{A}x = 4$
$Ay = 0$

the y=0
at any point
located on the exact x-axis.

$Bx = 12\cos 135 = -6\sqrt{2}\ m$
$By = 12\sin 135 = 6\sqrt{2}\ m$

$Rx = 4 + 0 - 6\sqrt{2} = -4.5$
$Ry = 0 - 6 + 6\sqrt{2} = 2.5$



$R = \sqrt{-4.5^2 + 2.5^2}$

$R = 5.1\ m$

$\theta = \tan^{-1}\left(\frac{2.5}{-4.5}\right)$

$\downarrow$

$\underline{\theta = 29°}$

Officially Done !! :)