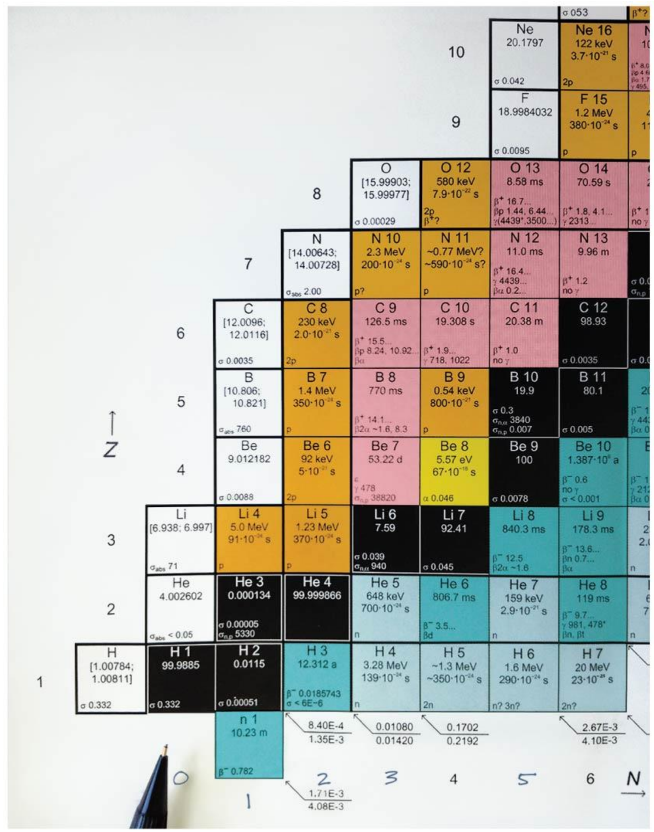
Chapter 30

Nuclear Physics and Radioactivity



30-1 Structure and Properties of the Nucleus

Nucleus is made of protons and neutrons

Proton has positive charge; here is its mass:

$$m_{\mathrm{p}} = 1.67262 \times 10^{-27}\ \mathrm{kg}$$

Neutron is electrically neutral, and slightly more massive than the proton:

$$m_{\mathrm{n}} = 1.67493 \times 10^{-27}\ \mathrm{kg}$$

30-1 Structure and Properties of the Nucleus

Neutrons and protons are collectively called nucleons.

The different nuclei are referred to as nuclides.

Number of protons: atomic number, Z

Number of nucleons: atomic mass number, A

Neutron number: $N = A - Z$

30-1 Structure and Properties of the Nucleus

A and Z are sufficient to specify a nuclide. Nuclides are symbolized as follows:

$$^{A}_{Z}\mathrm{X},$$

X is the chemical symbol for the element; it contains the same information as *Z* but in a more easily recognizable form.

30-1 Structure and Properties of the Nucleus

Nuclei with the same *Z*—so they are the same element—but different *N* are called isotopes.

For many elements, several different isotopes exist in nature.

Natural abundance is the percentage of a particular element that consists of a particular isotope in nature.

30-1 Structure and Properties of the Nucleus

Because of wave-particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx (1.2 \times 10^{-15}\ \text{m})\left(A^{\frac{1}{3}}\right). \quad (30\text{-}1)$$

4. (II) (*a*) What is the approximate radius of a $^{112}_{48}\text{Cd}$ nucleus? (*b*) Approximately what is the value of A for a nucleus whose radius is 3.7×10^{-15} m?

b) $3.7 \times 10^{-15} = 1.2 \times 10^{-15}\sqrt[3]{A}$

$A \approx 30$

a) $r \approx 12 \times 10^{-16}\sqrt[3]{A}$

$r \approx 1111\sqrt[3]{112}$

$r \approx 5.7 \times 10^{-15}$

30-1 Structure and Properties of the Nucleus

Masses of atoms are measured with reference to the carbon-12 atom, which is assigned a mass of exactly 12u. A u is a unified atomic mass unit.

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$$

30-1 Structure and Properties of the Nucleus

From the following table, you can see that the electron is considerably less massive than a nucleon.

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

Object	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^{1}_{1}$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

30-3 Radioactivity

Towards the end of the 19th century, minerals were found that would darken a photographic plate even in the absence of light.

This phenomenon is now called radioactivity.

Marie and Pierre Curie isolated two new elements that were highly radioactive; they are now called polonium and radium.

30-3 Radioactivity

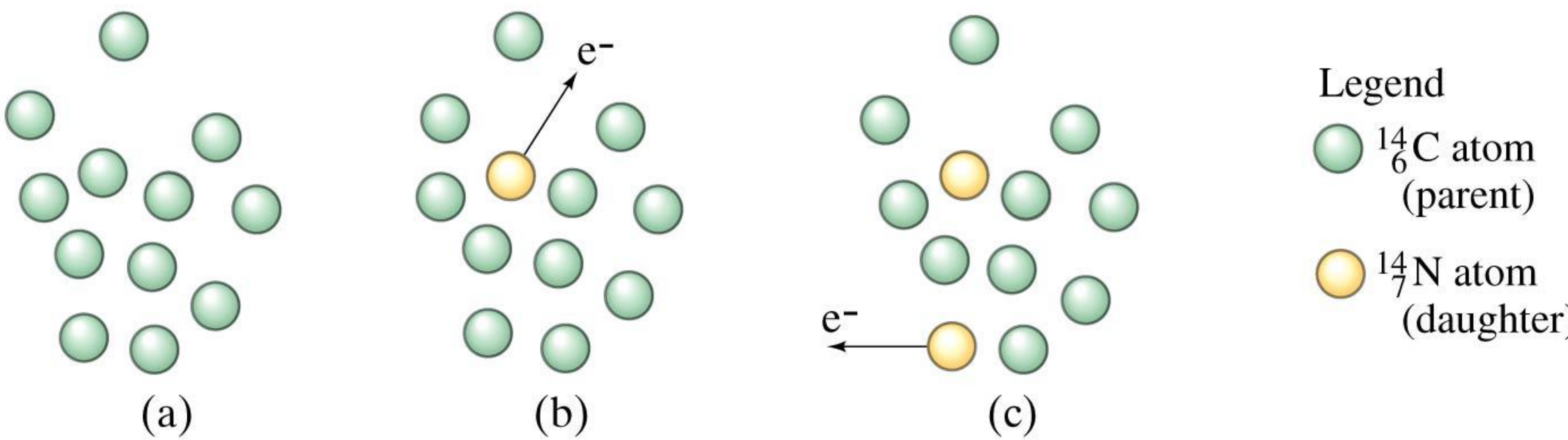
Radioactive rays were observed to be of three types:

1. Alpha rays, which could barely penetrate a piece of paper
2. Beta rays, which could penetrate 3 mm of aluminum
3. Gamma rays, which could penetrate several centimeters of lead

We now know that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are electromagnetic radiation.

30-8 Half-Life and Rate of Decay

Nuclear decay is a random process; the decay of any nucleus is not influenced by the decay of any other.



30-8 Half-Life and Rate of Decay

Therefore, the number of decays in a short time interval is proportional to the number of nuclei present and to the time:

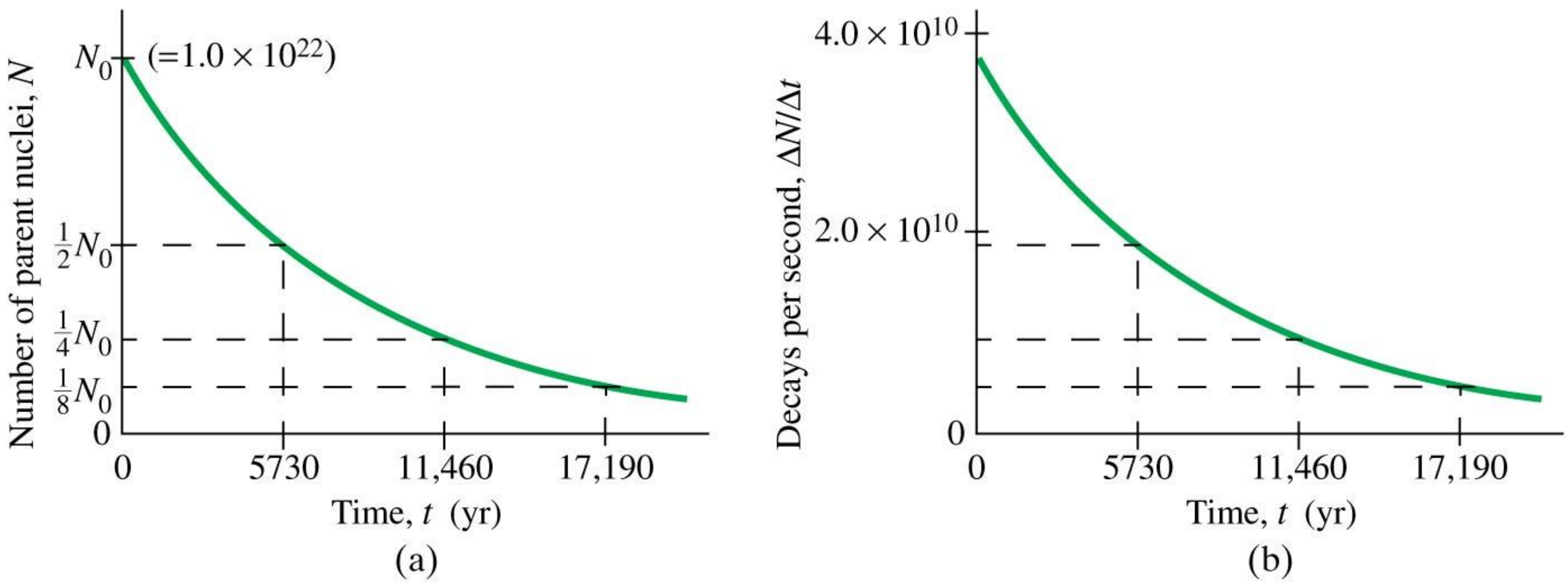
$$\Delta N = -\lambda N \, \Delta t \quad (30\text{-}3a)$$

Here, λ is a constant characteristic of that particular nuclide, called the decay constant.

30-8 Half-Life and Rate of Decay

This equation can be solved, using calculus, for *N* as a function of time:

$$N = N_0 e^{-\lambda t}, \quad (30\text{-}4)$$



37. (I) (*a*) What is the decay constant of $^{238}_{92}U$ whose half-life is 4.5×10^9 yr? (*b*) The decay constant of a given nucleus is $3.2 \times 10^{-5}\ s^{-1}$. What is its half-life?

$t_{\frac{1}{2}} = 45 \times 10^8 * 365 * 24 * 60 * 60$ s

$t_{\frac{1}{2}} = 14 * 10^{88}$ sec

$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = 4.9 * 10^{-9}$

b) $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$t_{\frac{1}{2}} = 0.21 * 10^5$ s

30-8 Half-Life and Rate of Decay

The half-life is the time it takes for half the nuclei in a given sample to decay. It is related to the decay constant:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \quad (30\text{-}6)$$

Activity

- The number of decays per second, or decay rate *R*, is the magnitude of and is also called the **activity** of the sample.

$$R = \left|\frac{\Delta N}{\Delta t}\right| = R_0 e^{-\lambda t},$$

where

$$\frac{\Delta N}{\Delta t} = \lambda N = \frac{0.693}{T_{\frac{1}{2}}} N.$$

The unit of activity is the number of disintegrations per second, often measured in curies, Ci.

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{disintegrations per second}$$

The SI unit for source activity is the becquerel (Bq):

$$1\ \text{Bq} = 1\ \text{disintegration/s}$$

EXAMPLE 30–9 **Sample activity.** The isotope ${}^{14}_{6}C$ has a half-life of 5730 yr. If a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

$t_{1/2}$ = 1.8×10^{11} s

N_0

$\left|\frac{\Delta N}{\Delta t}\right| = N\lambda = \frac{N \cdot \ln 2}{t_{1/2}} = \frac{10^{22} \cdot \ln 2}{1.8 \times 10^{11}} = 3.83 \times 10^{10}\ s^{-1}$

$\frac{\Delta N}{\Delta t} / R$

49. (II) The activity of a sample drops by a factor of 6.0 in 9.4 minutes. What is its half-life?

$t = 9.4$ $\quad R = \frac{R_0}{6}$

$R = R_0 e^{-\lambda t}$

$\frac{R_0}{6} = R_0 e^{-\lambda t}$

$e^{-\lambda (9.4)} = \frac{1}{6}$

$9.4 \times -\lambda = \ln\left(\frac{1}{6}\right)$

$-\frac{\ln 2}{t_{1/2}} = \frac{\ln\left(\frac{1}{6}\right)}{9.4}$

$t_{1/2} = 3.6$ min

Mean Life

- The mean life of an isotope is then given by

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693}$$

EXAMPLE 30–11 **A sample of radioactive $^{13}_{7}N$.** A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min (600 s). (*a*) How many nuclei are present initially? (*b*) What is the rate of decay (activity) initially? (*c*) What is the activity after 1.00 h? (*d*) After approximately how long will the activity drop to less than one per second $(= 1\ s^{-1})$?

$\frac{13g}{mol} = Mr$

a) $13g \to 1 mol \to 6.023 \times 10^{23}$ atoms (nuclei)

$13 \to 6.023 \times 10^{23}$

$1.49 \times 10^{-6} \to X$ nuclei

$N_0 = 6.9 \times 10^{16}$

b) $R = N_0 \lambda$

$R = \frac{6.9 \times 10^{16} \times \ln 2}{600} = 7.9 \times 10^{13}\ s^{-1}$

c) $T = 3600 sec$

$T = 6 \times 600 = 6 t_{\frac{1}{2}}$

$R = \left(\frac{1}{2}\right)^6 R_o$

$R = \frac{1}{64} \times 7.9 \times 10^{13} = 1.25 \times 10^{12}$

$\frac{\Delta N}{\Delta t} = 1$

$R = R_o e^{-\lambda t}$

$1 = 7.9 \times 10^{13} \times e^{-1.15 \times 10^{-3} t}$

$t = 2.77 \times 10^{4}\ s$

30-11 Radioactive Dating

Radioactive dating can be done by analyzing the fraction of carbon in organic material that is carbon-14.

The ratio of carbon-14 to carbon-12 in the atmosphere has been roughly constant over thousands of years. A living plant or tree will be constantly exchanging carbon with the atmosphere, and will have the same carbon ratio in its tissues.

30-11 Radioactive Dating

When the plant dies, this exchange stops. Carbon-14 has a half-life of about 5730 years; it gradually decays away and becomes a smaller and smaller fraction of the total carbon in the plant tissue.

This fraction can be measured, and the age of the tissue deduced.

Objects older than about 60,000 years cannot be dated this way—there is too little carbon-14 left.

30-11 Radioactive Dating

Other isotopes are useful for geologic time scale dating.

Uranium-238 has a half-life of 4.5×10^9 years, and has been used to date the oldest rocks on Earth as about 4 billion years old.

$\lambda = 3.83 * 10{-}12\ s{-}$

EXAMPLE 30–13 **An ancient animal.** The mass of carbon in an animal bone fragment found in an archeological site is 200 g. If the bone registers an activity of 16 decays/s, what is its age?

$\frac{{}^{14}_{6}C}{{}^{12}_{6}C} = 1.3 * 10^{-12}$

* الـ 200 g من الكربون تقريبا كلهم من نوع ${}^{12}_{6}C$

$1 \longrightarrow 6.023 * 10^{23}$

$16.6 \longrightarrow$ عدد نوى ${}^{12}_{6}C$

$10^{25} =$ عدد نوى ${}^{12}_{6}C$ $=$ عدد ذرات ${}^{12}_{6}C$

$N_0 = {}^{14}C = (10^{25})(1.3 * 10^{-12})$

$N_0 = 1.3 * 10^{13}$

$R_0 = \lambda N_0$

$R_0 = (1.3 * 10^{13})(3.83 * 10^{-12})$

$R_0 = 50\ s^{-1}$

$R = R_0 e^{-\lambda t}$

$16 = 50 e^{-3.83 * 10^{-12} t}$

$t = 2.98 * 10^{12}$

42. (II) The iodine isotope $^{131}_{53}I$ is used in hospitals for diagnosis of thyroid function. If 782 μg are ingested by a patient, determine the activity (*a*) immediately, (*b*) 1.50 h later when the thyroid is being tested, and (*c*) 3.0 months later. Use Appendix B.

$t_{1/2} = (8.02)(24)(60)(60)$

(a) $1 \rightarrow 6.023 \times 10^{23}$

$\frac{782 \times 10^{-6}}{131} \rightarrow N_0$

$N_0 = 3.59 \times 10^{18}$

$\left.\frac{\Delta N}{\Delta t}\right|_0 = N_0 e^{-\lambda(0)}$, where $e^{-\lambda(0)} = 1$

$\left.\frac{\Delta N}{\Delta t}\right|_0 = 3.59 \times 10^{18}$

$R_0 = N_0 = 3.59 \times 10^{18}$

$R = R_0 e^{-\lambda t}$

$\lambda = 10^{-6}$

$R = 3.57 \times 10^{12}$

43. (II) How many nuclei of $^{238}_{92}U$ remain in a rock if the activity registers 420 decays per second?

$t_{\frac{1}{2}} = 4.5 \times 10^9 y$

$$\frac{\Delta N}{\Delta t} = N\lambda$$

$$N = \frac{420 \times 4.5 \times 10^9 \times 365 \times 24 \times 3600}{\ln 2}$$

$N = 8.69 \times 10^{19}$ nuclei

46. (II) Calculate the mass of a sample of pure $^{40}_{19}K$ with an initial decay rate of $2.4 \times 10^5\ s^{-1}$. The half-life of $^{40}_{19}K$ is 1.248×10^9 yr.

we need (N_0)

$$\left|\frac{\Delta N}{\Delta t}\right|_0 = N_0 * \frac{\ln 2}{t_{\frac{1}{2}}}$$

$$N_0 = 1.36 * 10^{22}$$

$$m = 0.99$$

$$1 \rightarrow 6.023 * 10^{23}$$

$$\frac{m}{40} \rightarrow N_0$$

$$m = \frac{N_0 * 40}{6.023 \times 10^{23}}$$