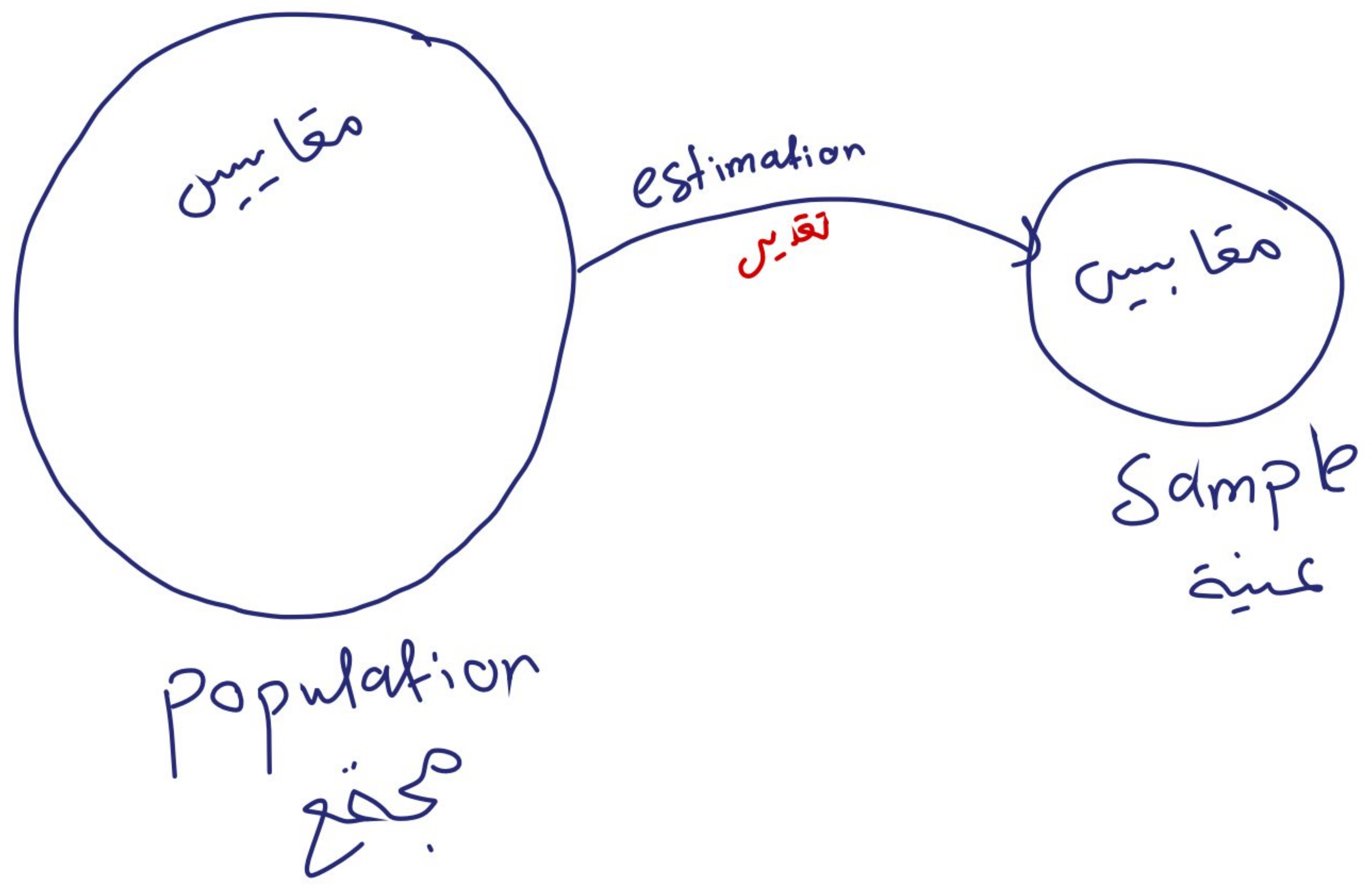
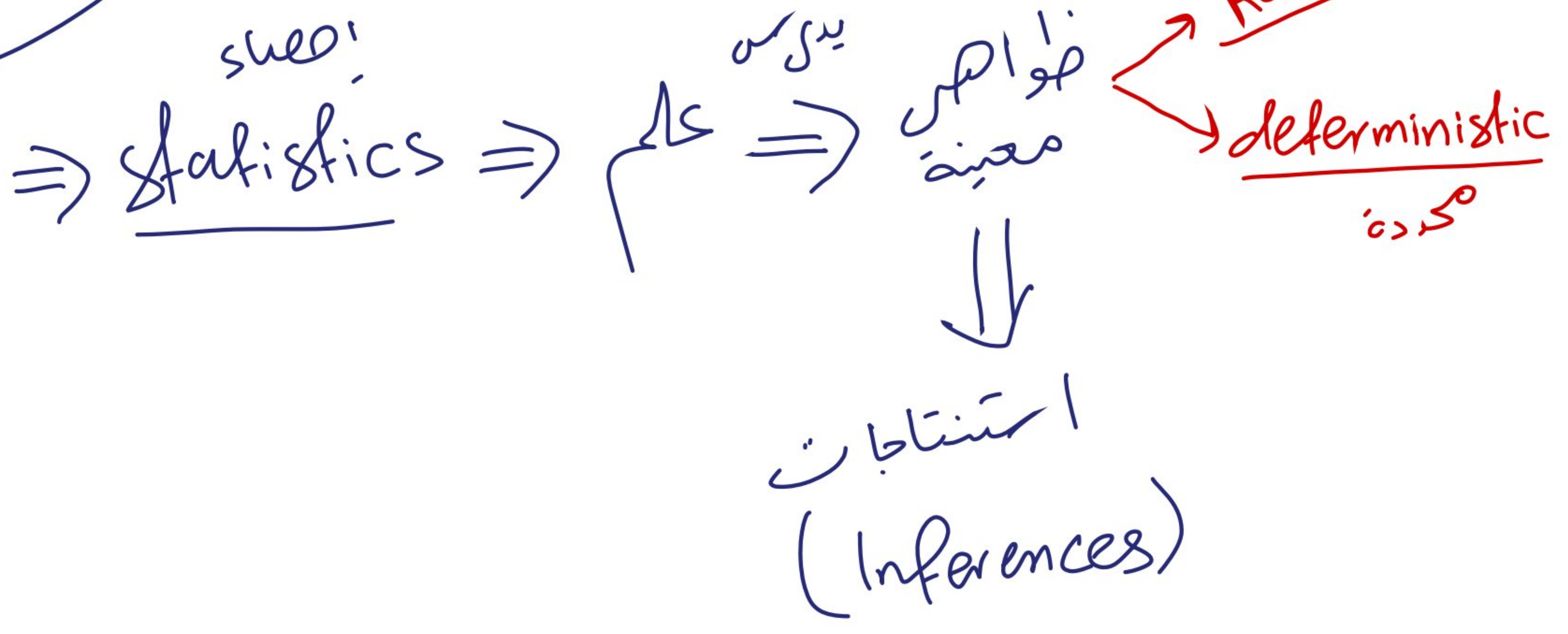




BioStatistics

"for medical students"

Introduction to Statistics



* Mathematical Statistics

↳ Branch

as lip Techniques + Statistical methods as lip

★ Applied statistics

↳ Application of statistical methods and Techniques

BIOSTATISTICS

↙
medicine

↘
Biology

البيانات
(Data)

question

→ Data collection

← جمع البيانات

→ Data entry

تحليل البيانات

Data Analysis

↓
Data editing

↓

Inferences → publication
الاستنتاجات

Data ⇒ Summarization
تلخيص

Descriptive statistics
الإحصاء الوصفي

Summarize
describe

Inferential statistics
الإحصاء الاستنتاجي

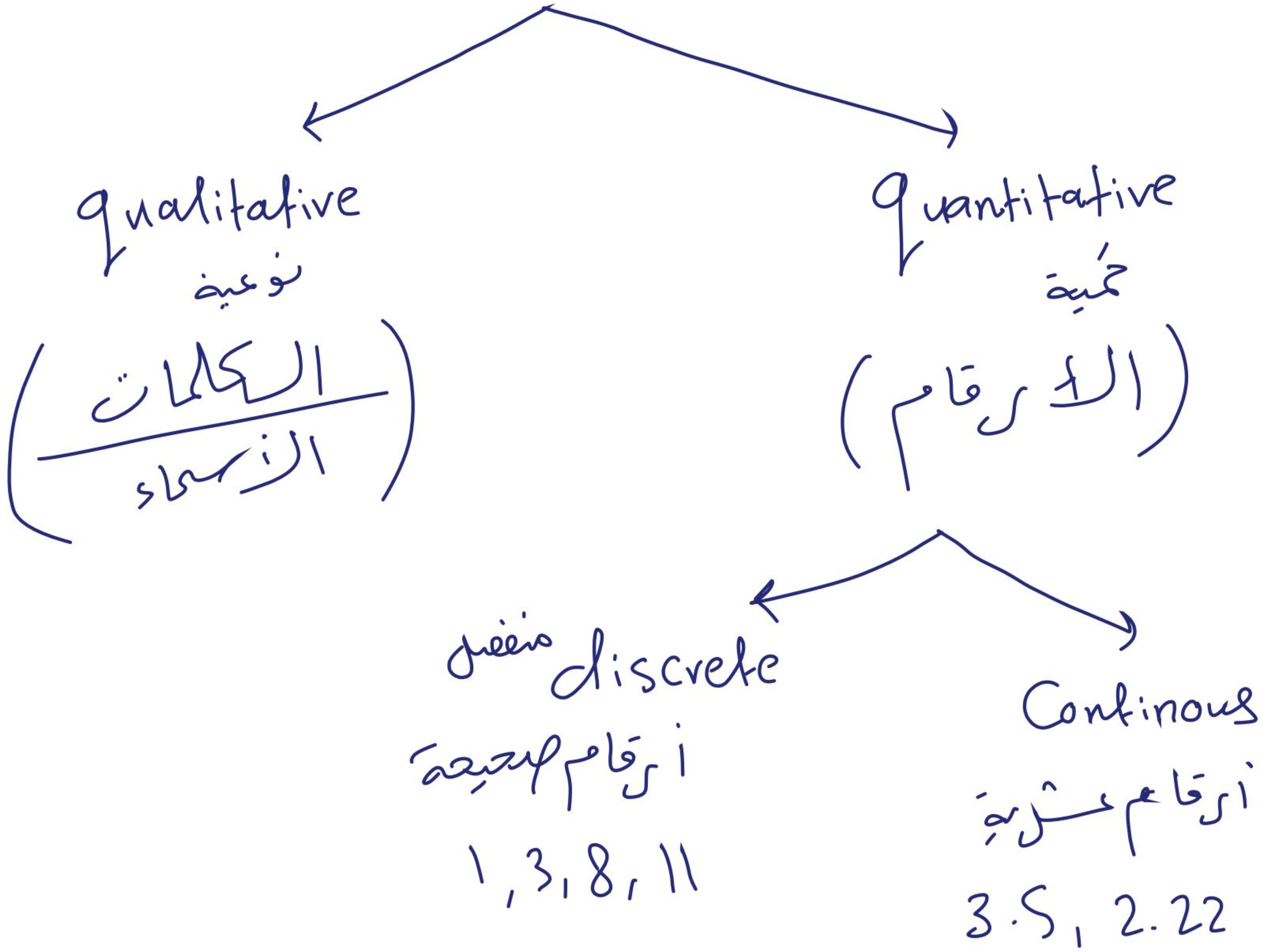
Conclusions
predictions

كيف نعرف البيانات؟

① numeric أرقام

② graphic رسومات

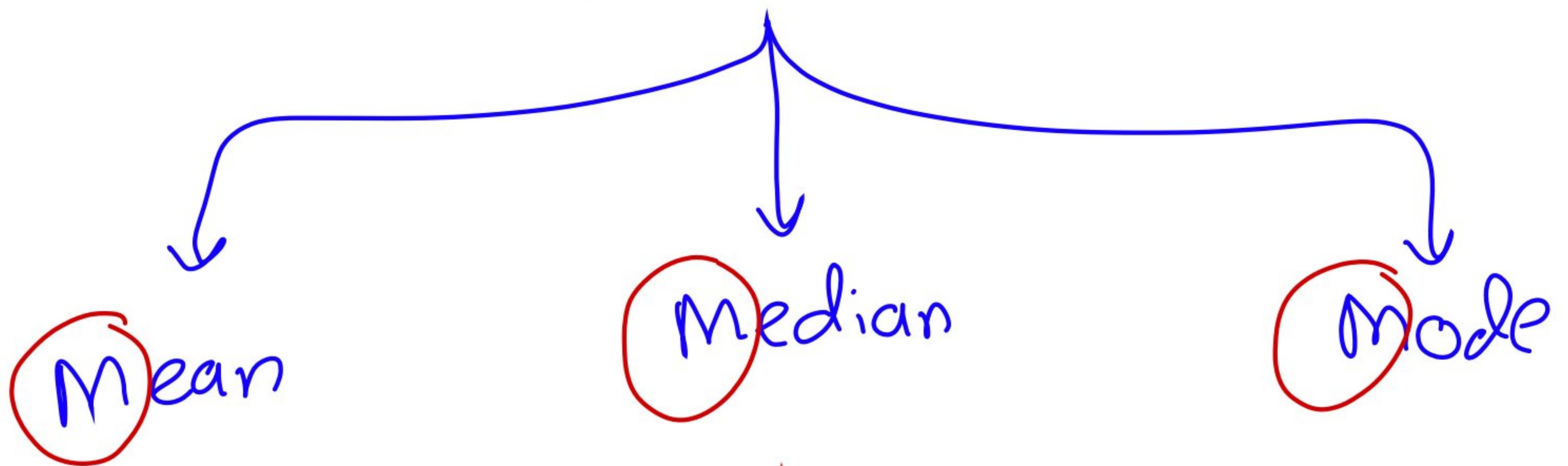
Data



Chapter 2

Descriptive Statistics

* Measures of Central tendency
مقاييس النزعة الوسطى



3 M's

① Mean (Arithmetic mean)

$$\bar{X} = \frac{\sum X}{n} = \frac{\text{مجموع البيانات}}{\text{عدد داتا}}$$

Example

find the mean for:

(2, 7, 5, 11, 5)

$$\bar{X} = \frac{2 + 7 + 5 + 11 + 5}{5} = 6$$

Example If the mean of $(a, a, 7, 11, 2)$ is 6, find the a ?

$$6 = \frac{a+a+7+11+2}{5}$$

$$30 = 2a + 20$$

$$2a = 30 - 20$$

$$2a = 10$$

$$a = 5$$

Example If the mean of $(x, y, 12)$ is 10, find the mean of (x, y) ?

$$10 = \frac{x+y+12}{3}$$

$$30 = x+y+12$$

$$x+y = 18$$

$$\frac{x+y}{2} = \frac{18}{2} = 9$$

Example find the mean of
(-9, -7, -11, 8, 2)

$$\bar{X} = \frac{-9 + -7 + -11 + 8 + 2}{5}$$

$$= -3.4$$

NOTE \bar{X} could be negative

$$\bar{X} = \frac{\sum X}{n}$$

$$\sum X = \bar{X} \cdot n$$

عندما يحدث تغيير على مجموع قيم X

معدل قيم X / معدل
طال يغير على معدل

Example If the mean mark of 9 students is 15. Ahmad with mark 20 joined the class, find the new mean?

~~84~~

$$n = 9$$

$$\bar{X} = \underline{\underline{15}}$$

$$X = 20$$

Ahmad

$$n = 10$$

$$\bar{X} = ?$$

$$\sum X_{\text{قديم}} = \bar{X} \cdot n$$

$$= 15 * 9 = 135$$

$$\bar{X}_{\text{new}} = \frac{\sum X_{\text{new}}}{n_{\text{new}}}$$

$$\bar{X}_{\text{new}} = \frac{\sum X_{\text{new}}}{10}$$

$$\sum X_{\text{جديد}} = 135 + 20 = 155$$

$$= \frac{155}{10}$$

$$\bar{X}_{\text{new}} = 15.5$$

Example If the mean mark of 10 Boys is 12, and the mean mark of 12 Girls is 10 find the mean mark of students altogether?

~~85~~

$$n = \underline{\underline{10}}$$

$$\bar{X} = 12$$

Boys

$$\sum X_B = \bar{X} \cdot n$$

$$= 12 * 10$$

$$= 120$$

$$n = \underline{\underline{12}}$$

$$\bar{X} = 10$$

Girls

$$\sum X_G = \bar{X} \cdot n$$

$$= 10 * 12$$

$$= 120$$

$$\bar{X}_{\text{total}} = \frac{\sum X_B + \sum X_G}{22} = \frac{120 + 120}{22} = 10.9$$

2) The mode "العنوان"
العنوان الأكثر تكراراً ←

Example find the mode for:

1) (2, 7, 5, 11, 5)

⇒ The mode is 5

2) (2, 7, 5, 11, 5, 2)

The mode is 5, 2

3) (2, 7, 5, 11, 5, 2, 5)

The mode is 5

NOTE

1 mode \Rightarrow unimodal

2 modes \Rightarrow Bimodal

3 modes \Rightarrow Trimodal

③ The median (Q_2)

① ولكن يتب

② نعد البيا ناً

زوجي

فردى

← نأخذ العتامة بالمتلن

2

← نأخذ العمة التي بالمتلن

Example

~~5~~, ~~5~~, ~~8~~, ~~10~~, 12, ~~12~~, ~~13~~, ~~13~~, ~~15~~

Find the median?

⇒ The median is 12

Example

2, 9, 11, 5, 6, 27

find Q_2 ?

الترتيب

~~2~~, ~~5~~, 6, 9, ~~11~~, ~~27~~

The median is $\frac{6+9}{2} = 7.5$

Example

2, 6, 16, 9, 3, 8, 11

الترتيب

~~2~~, ~~3~~, ~~6~~, 8, ~~9~~, ~~11~~, ~~16~~

The median is 8

Example ~~5, 7, 7, 9, 10, 10, 12, 14~~

The median is $\frac{9+10}{2}$
 $= 9.5$

Example For the following ORDERED DATA

(~~a~~, ~~2~~, ~~3~~, 5, b, ~~9~~, ~~10~~, ~~c~~). IF the mean is 7, the median is 6 and the mode is 2 Find the a, b, c?

$$\begin{aligned}\bar{x} &= 7 \\ Q_2 &= \underline{6} \\ \text{mode} &= 2\end{aligned}$$

~~3)~~

$$\text{from mode} \Rightarrow \boxed{a = 2}$$

$$\text{from median} \Rightarrow \frac{5+b}{2} = \frac{6}{1}$$

$$\boxed{b = 7}$$

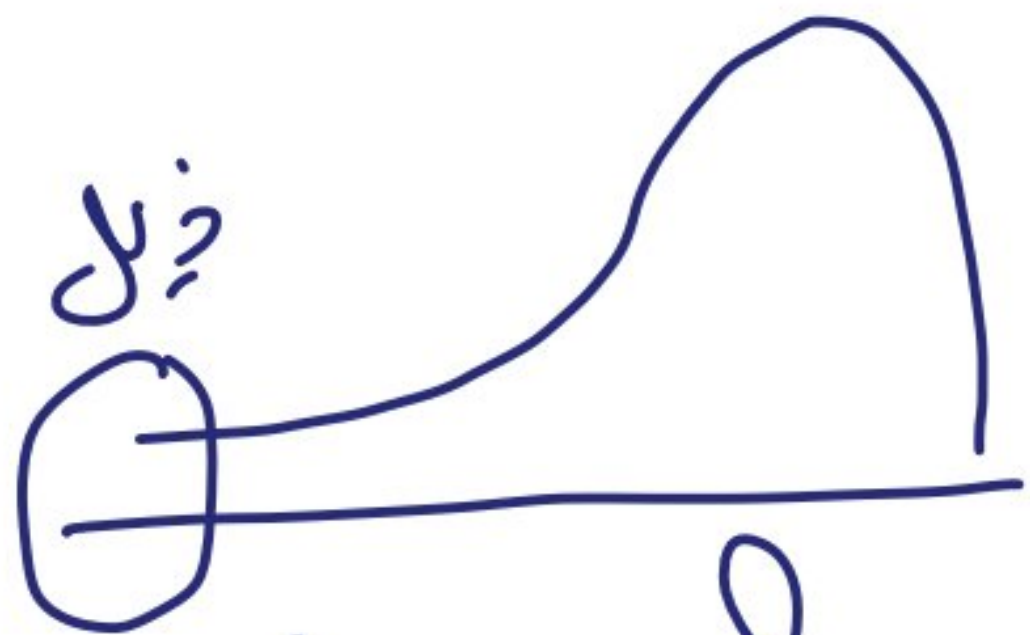
from mean \Rightarrow

$$7 = \frac{2+2+3+5+7+9+10+c}{8}$$

$$\boxed{c = 18}$$

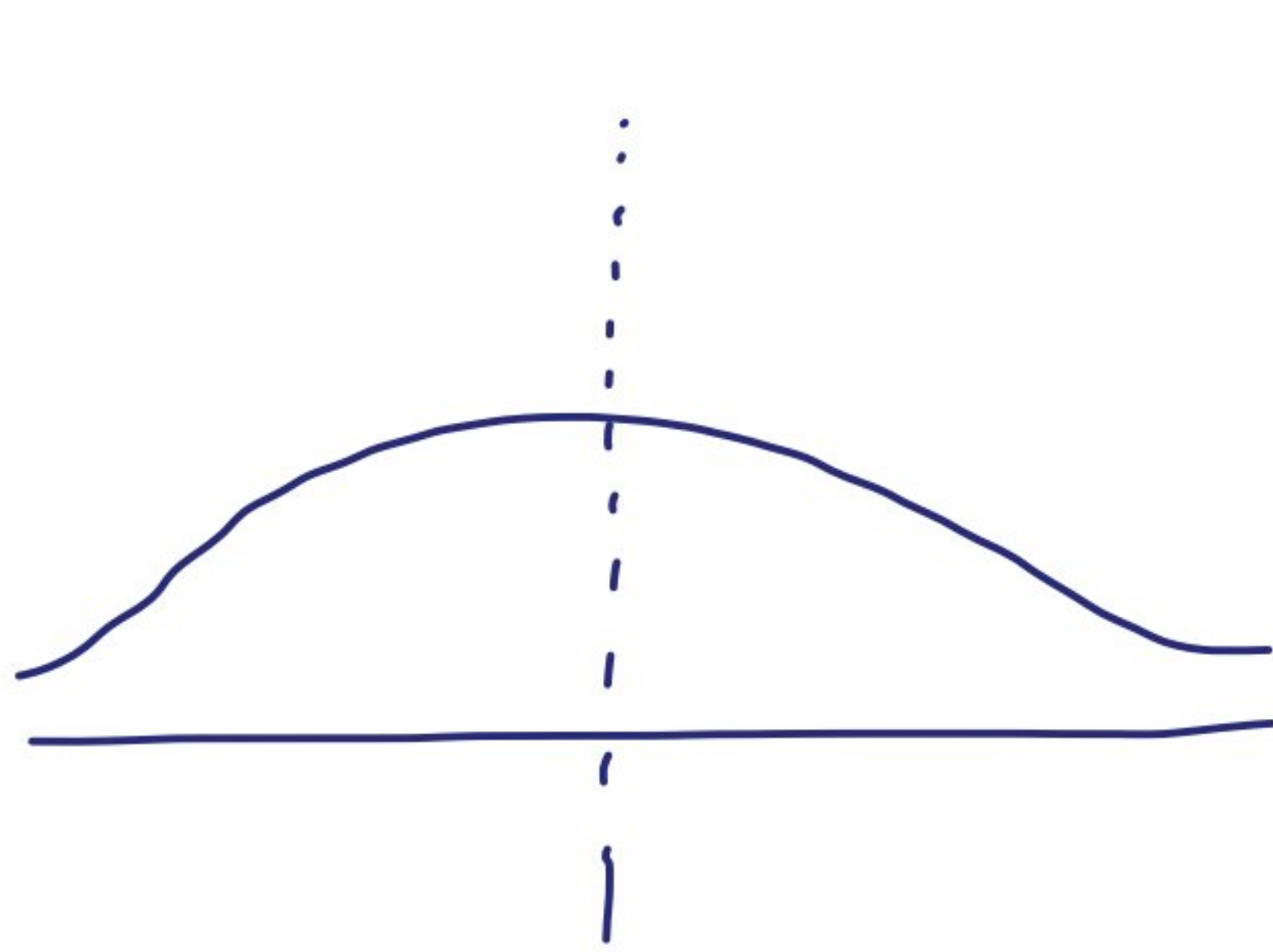
Skewness

اسکوز



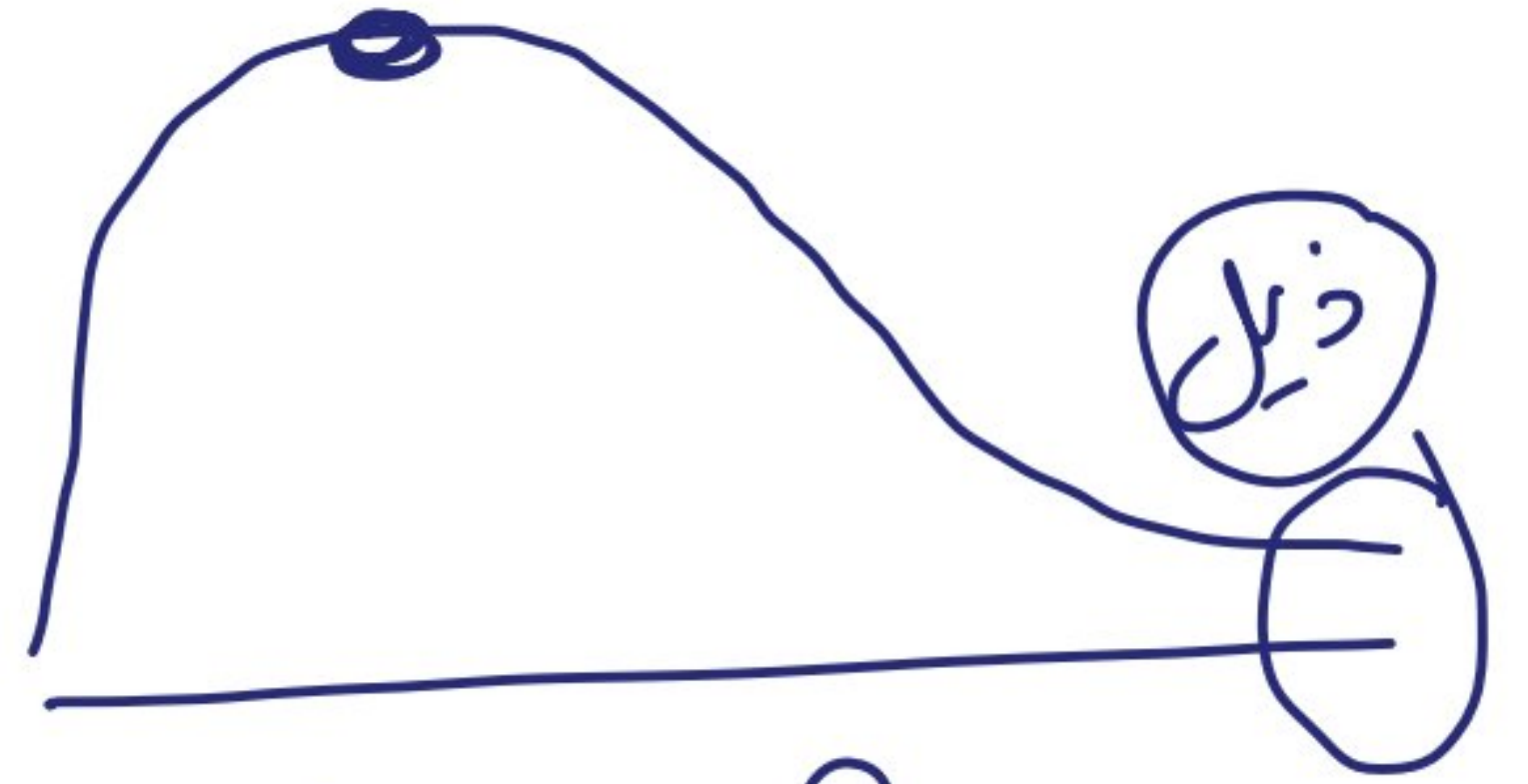
Skewed
to
Left

$$\bar{x} < Q_2 < \text{mode}$$



Symmetric

$$\text{mode} = Q_2 = \bar{x}$$



Skewed
to
Right

$$\text{mode} < Q_2 < \bar{x}$$

Example

Determine the shape of
the following:

1) mode = 16, $\bar{x} = 14$
mode > \bar{x} skewed to left

2) mode = 7630, $\bar{x} = 7630$
mode = mean Symmetric

3) mode = 2, $\bar{x} = 4$
 mean > mode skewed to Right

Characteristics of mean

① mean is affected by an outlier

outlier العَمَلِيَّة

2, 2, 2, 2, 20

outlier

2, 2, 2, 2 $\bar{x} = 2$

~~outlier~~

② 19, 19, 18, 20

2, 2, 2, 2, 20

$$\bar{x} = \frac{2+2+2+2+20}{5}$$

$$= 5.6$$

2) ③ 2, 2, 2, 2
 5, 5, 5, 5

$\bar{x} = 2$
 $\bar{y} = 5$
 +3

$$\bar{y} = \bar{x} + b$$

③ $2, 2, 2, 2 \quad \bar{x} = 2$
 $\downarrow \times 3 \quad \downarrow \times 3$
 $6, 6, 6, 6 \quad \bar{y} = 6$

* Measures of Variability (Spread)

مقاييس التباين

مستحيل
يكونوا
سوال

① The Range = Max - Min
المدى

Example find the range for:

1) (2, 7, 5, 11, 2)

The Range is = $11 - 2 = 9$

② The variance and standard deviation

مقاييس التباين

$$① S^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$② S^2 = \frac{\sum x^2}{n - 1} - \frac{(\sum x)^2}{n(n - 1)}$$

$$S = \sqrt{\text{variance}} = \sqrt{\frac{\sum x^2}{n - 1} - \frac{(\sum x)^2}{n(n - 1)}}$$

$$\sum x^2 \quad \sqrt{\sum (x)^2}$$

مجموع القيم ثم
مربعها

2, 2, 2, 2
4, 4, 4, 4 $\sum x^2 = 16$

مجموع القيم ثم
مربع

2, 2, 2, 2
 $(\sum x)^2 = (8)^2 = 64$

Example find variance for (2, 7, 5, 11, 5)

$$\bar{x} = \frac{2 + 7 + 5 + 11 + 5}{5}$$

طريقة (1)

$$S^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$= 6$$

x	2	7	5	11	5
(x - \bar{x})	-4	1	-1	5	-1
(x - \bar{x}) ²	16	1	1	25	1

Always

$$\sum (x - \bar{x}) = 0$$

$$\sum (x - \bar{x})^2 = 44$$

$$S^2 = \frac{44}{5-1} = \frac{44}{4} = 11$$

$$S = \sqrt{11} = 3.31$$

المعادلة
(2)

$$S^2 = \frac{\sum X^2}{n-1} - \frac{(\sum X)^2}{n(n-1)}$$

X	2	7	5	11	5	$\sum X = 30$
X^2	4	49	25	121	25	$\sum X^2 = 224$

$$S^2 = \frac{224}{5-1} - \frac{(30)^2}{5(5-1)} = 11$$

$$S = \sqrt{11} = 3.31$$

Example

find the variance for (5, 7, 1, 2, 4)

(1) المعادلة

$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1}$$

$$\bar{X} = 3.8$$

X	5	7	1	2	4	
$(X - \bar{X})$	1.2	3.2	-2.8	-1.8	0.2	
$(X - \bar{X})^2$	1.44	10.24	7.84	3.24	0.04	$\sum (X - \bar{X})^2 = 22.8$

$$S^2 = \frac{22.8}{5-1} = 5.7$$

$$S = \sqrt{5.7} = 2.38$$

طريقة (2)

$$S^2 = \frac{\sum X^2}{n-1} - \frac{(\sum X)^2}{n(n-1)}$$

X	5	7	1	2	4	$\sum X = 19$
X ²	25	49	1	4	16	$\sum X^2 = 95$

$$S^2 = \frac{95}{5} - \frac{(19)^2}{5(4)} = 5.7$$

$$S = \sqrt{5.7} = 2.38$$

* Characteristics of variance

① مستحيل يكون سالب

②

2, 2, 2, 2 $\sum x^2 = a$

-2, +3

5, 5, 5, 5 $\sum y^2 = a$

لا يتأثر بالجمع والطرح

3

2, 2, 2, 2
*3
6, 6, 6, 6

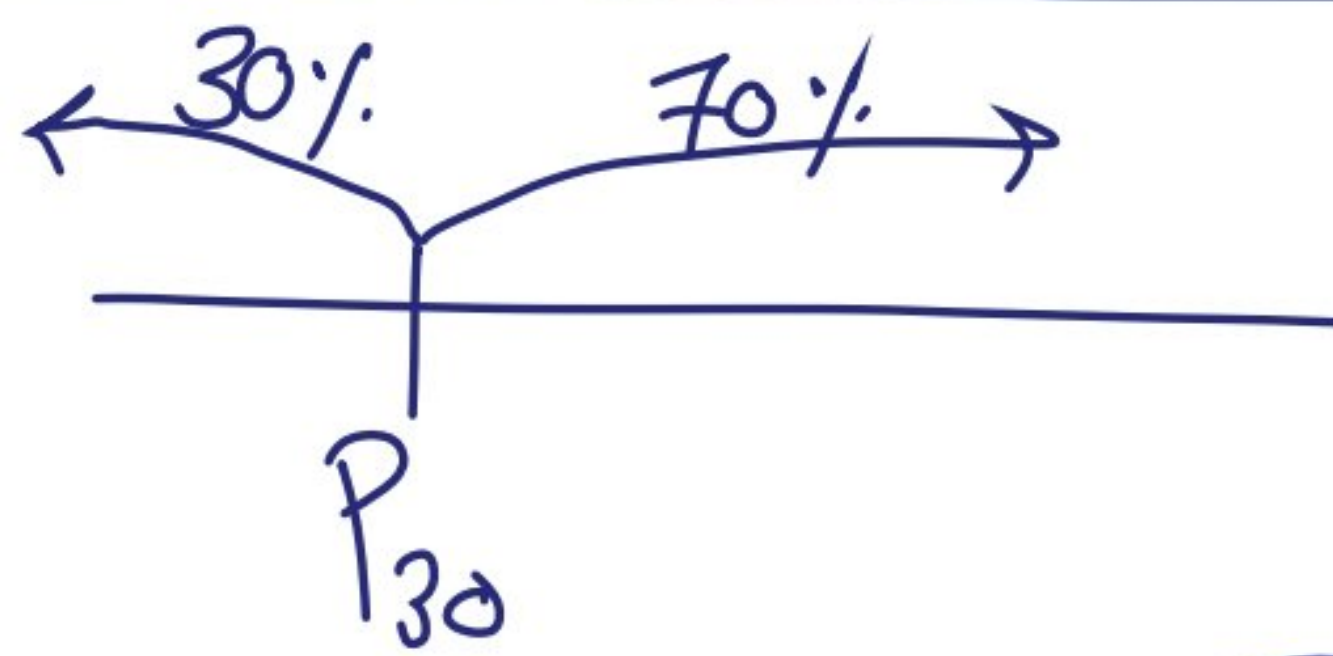
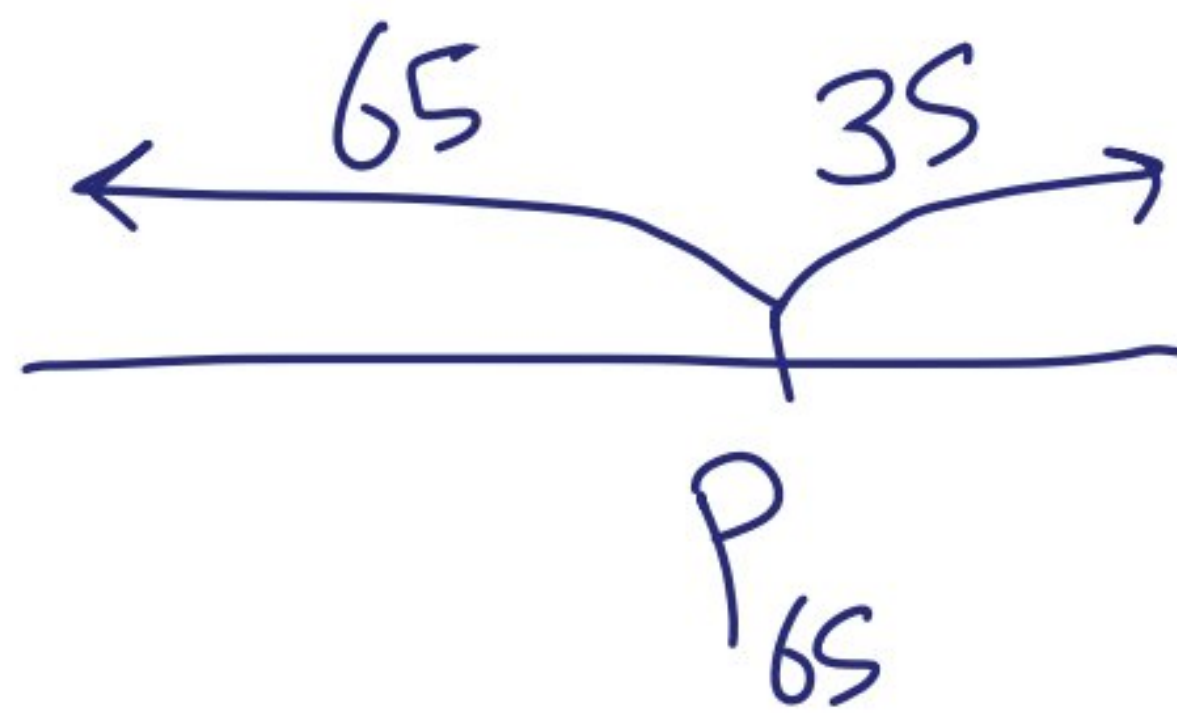
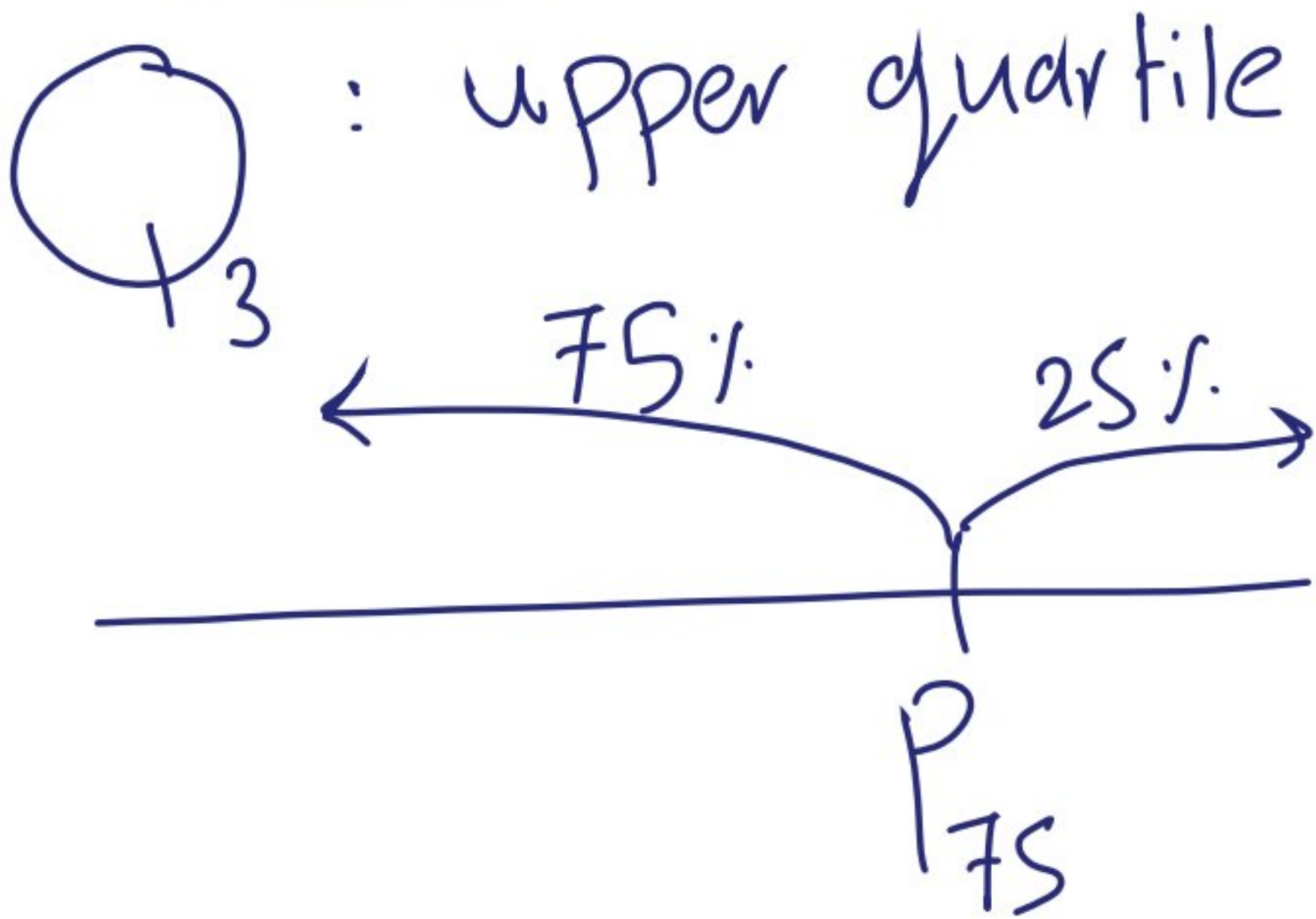
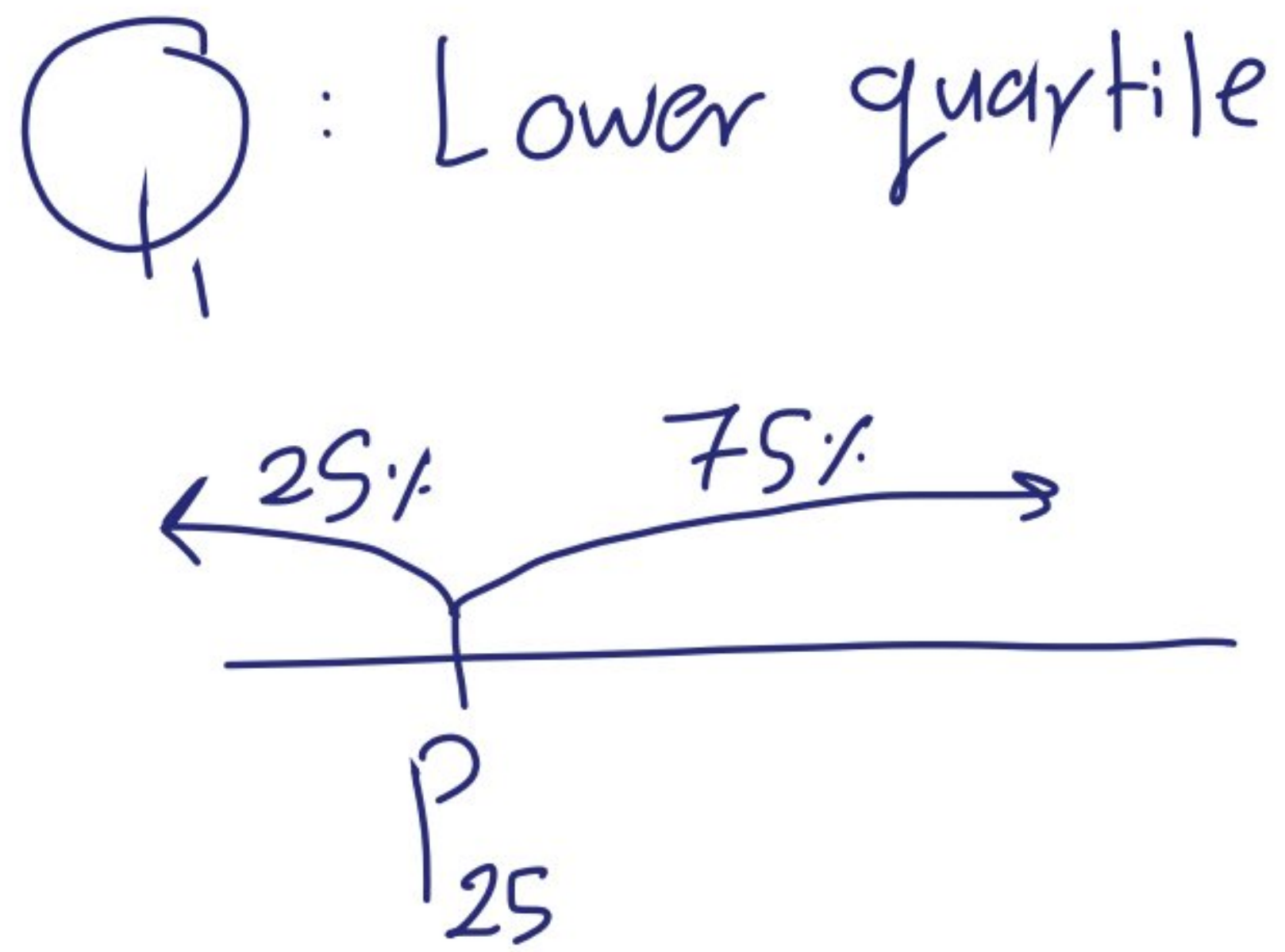
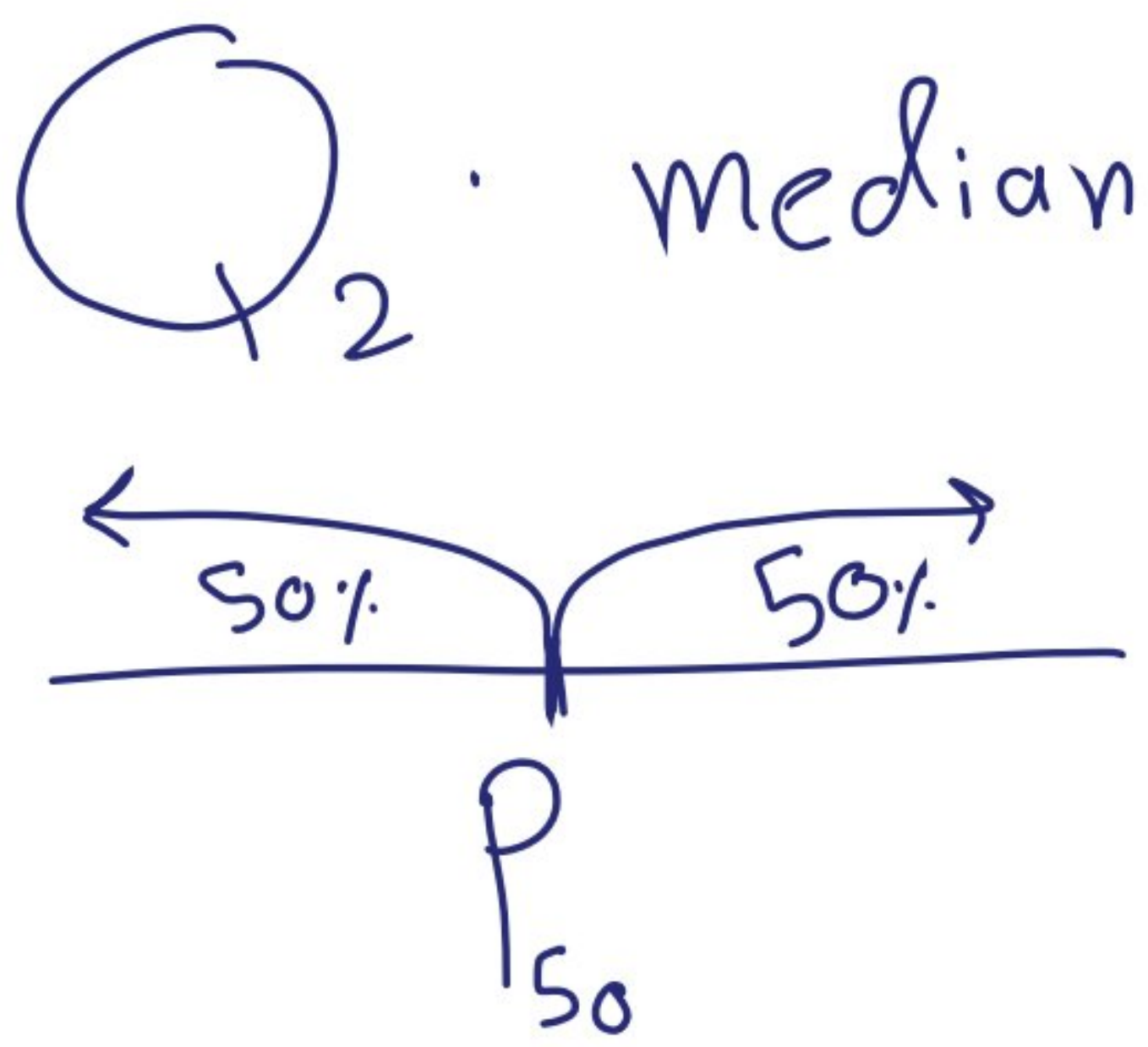
$$\sum x^2 = a$$

$\sum x^2$: Variance (التباين)
 $\sum y^2$: Variance (التباين)

$$\sum y^2 = C^2 * \sum x^2$$

$$\sum y^2 = 9 * a$$

★ Quartiles and Percentiles



Percentile P_p

$$P_p = \frac{n \cdot P}{100}$$

n: Sample Size
P: Percentile

→ next integer

عدد صحيح

$$2.2 \Rightarrow (3)^{\text{th}}$$

$$3.9 \Rightarrow (4)^{\text{th}}$$

$$\frac{\text{العدد} + (\text{العدد} + 1)}{2}$$

$$\frac{(3)^{\text{th}} + (4)^{\text{th}}}{2}$$

$$\frac{(5)^{\text{th}} + (6)^{\text{th}}}{2}$$

Inter-quartile Range (IQR)

$$\text{IQR} = Q_3 - Q_1$$

Example 2, 7, 5, 11, 3, 8, 10 Find IQR:

الترتيب \rightarrow 2, 3, 5, 7, 8, 10, 11

$$Q_1 = P_{25} = \frac{n \cdot P}{100} = \frac{7 \times 25}{100} = \frac{175}{100} = (1.75)^{\text{th}} = (2)^{\text{th}}$$

$$Q_1 = 3$$

$$Q_3 = P_{75} = \frac{7 \times 75}{100} = \frac{525}{100} = (5.25)^{\text{th}} = (6)^{\text{th}}$$

$$Q_3 = 10$$

$$IQR = 10 - 3 = 7$$

Example

260, 290, 300, 320, 330, 340, 340, 550
find IQR?

$$Q_1 = P_{25} = \frac{8 \times 25}{100} = \frac{200}{100} = \frac{(2)^{\text{th}} + (3)^{\text{th}}}{2}$$

$$\frac{290 + 300}{2} = 295$$

$$Q_3 = P_{75} = \frac{8 \times 75}{100} = \frac{600}{100} = \frac{(6)^{\text{th}} + (7)^{\text{th}}}{2}$$

$$\frac{340 + 340}{2} = 340$$

$$IQR = 340 - 295 = 45$$

Example

2, 3, 5, 7, 9, 10, 11, 12 Find

the value that 65% of data lie below it?

$$P_{65} = \frac{8 \times 65}{100} = \frac{520}{100} = (5.2)^{\text{th}} = (6)^{\text{th}}$$

$$P_{65} = 10$$

Example

3, 5, 5, 6, 8, 8, 9, 10, 11, 12

find the value that 30% of data lie above it?

$$P_{70} = \frac{10 \times 70}{100} = \frac{(7)^{\text{th}} + (8)^{\text{th}}}{2} = \frac{9 + 10}{2} = 9.5$$

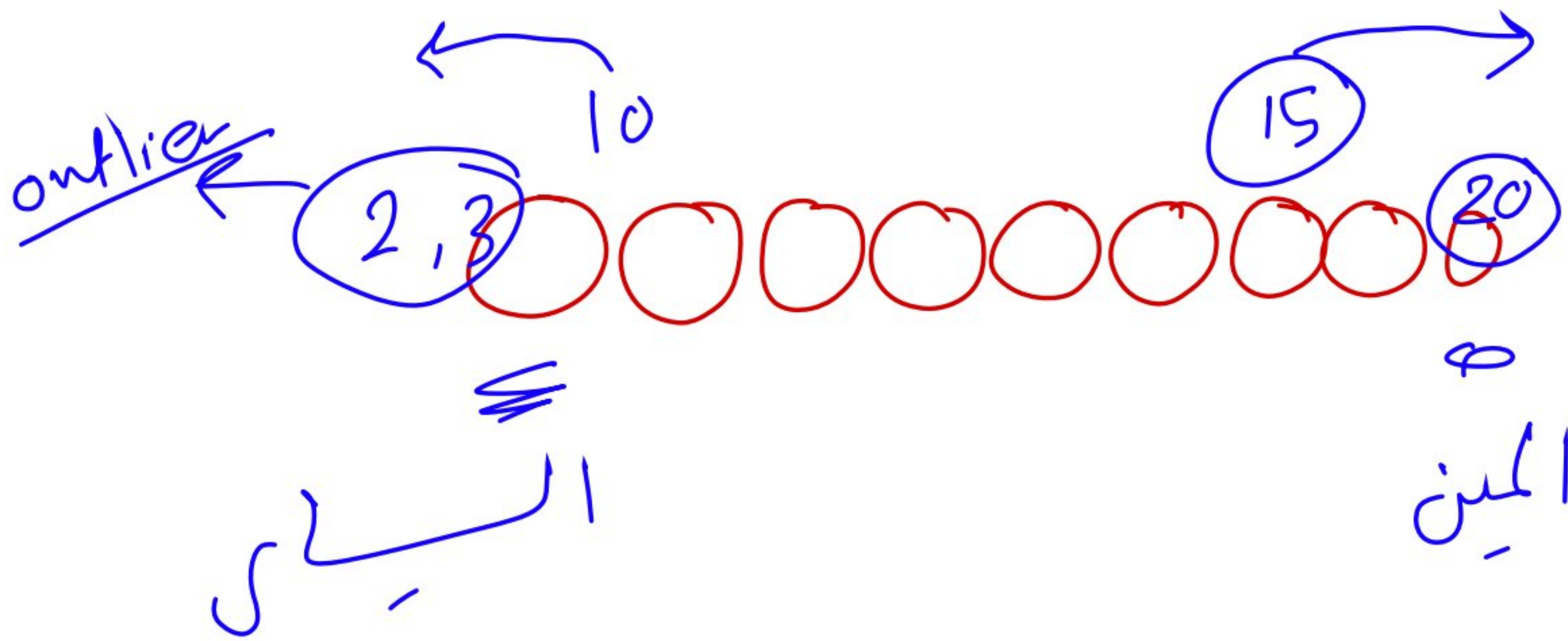
Outliers
 العنصر المتأخر
مفرد

مؤهل

$$Q_3 + 1.5 * IQR$$

من المئين

$$Q_1 - 1.5 * IQR$$



Example 123, 140, 145, 146, 147, 149, 150, 172

find an outliers (if there was)
 ~! ~! ~!

من المئين

$$Q_3 + 1.5 * IQR$$

$$149.5 + 1.5 * 7 = 160$$

∴ 172 is an outlier

من المئين

$$Q_1 - 1.5 * IQR$$

$$142.5 - 1.5 * 7 = 132$$

∴ 123 is an outlier

$$Q_1 : P_{25} = \frac{8 * 25}{100} = \frac{(2)^{th} + (3)^{th}}{2} = \underline{140 + 145}$$

$$Q_3 : P_{75} = \frac{8 \times 75}{100} = \frac{(6)^{th} + (7)^{th}}{2}$$

$$= \frac{149 + 150}{2}$$

$$= 149.5$$

$$IQR = 149.5 - 142.5 = 7$$

Example

340, 300, 520, 340, 320, 290, 260, 330
 find outliers (if there was) ?

من الحدين

$$Q_3 + 1.5 * IQR$$

$$340 + 1.5 * 45 = 407.5$$

∴ 520 is an outlier

من الحد الآخر

$$Q_1 - 1.5 * IQR$$

$$295 - 1.5 * 45 = 227.5$$

X

الترتيب 2275

260, 290, 300, 320, 330, 340, 340, 340, 520

4075

$$Q_1 : P_{25} = \frac{8 \times 25}{100} = \frac{(2)^{\text{th}} + (3)^{\text{th}}}{2}$$

$$= \frac{290 + 300}{2}$$

$$= 295$$

$$Q_3 : P_{75} = \frac{8 \times 75}{100} = \frac{(6)^{\text{th}} + (7)^{\text{th}}}{2}$$

$$= \frac{340 + 340}{2}$$

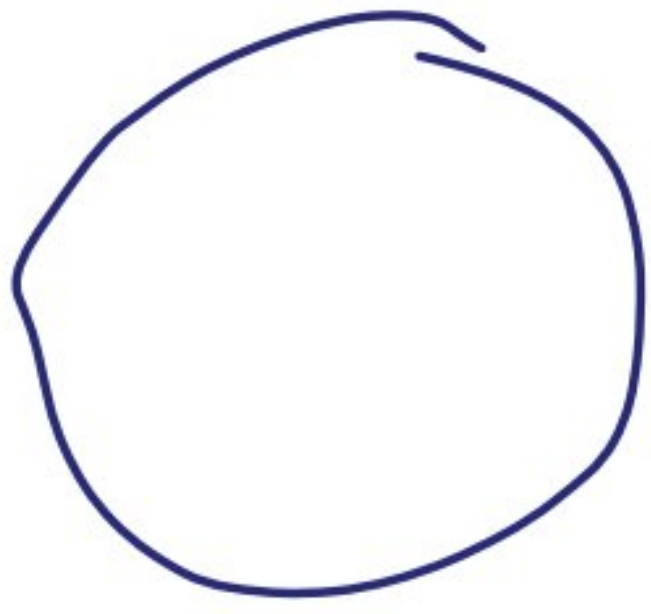
$$= 340$$

$$IQR = 340 - 295 = 45$$

★ Coefficient of variation

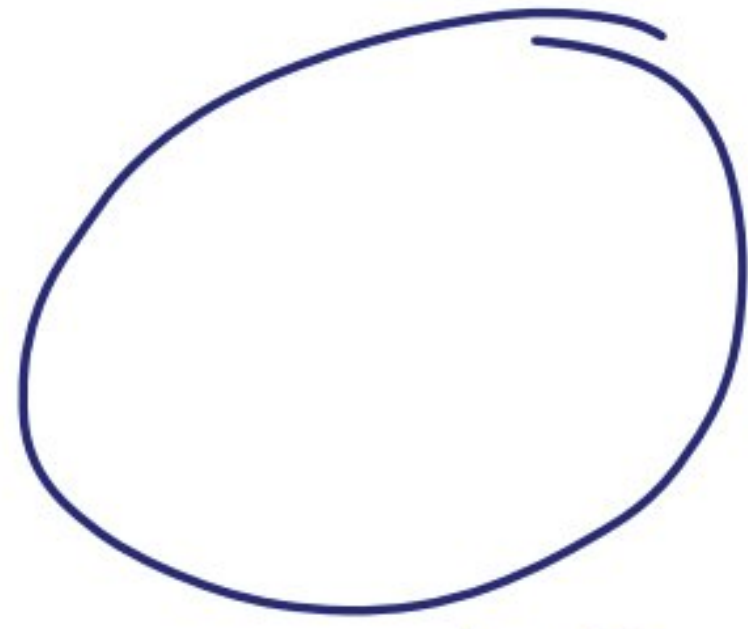
(CV)

variation



دولاره

$$S = 0 (\$)$$



دولاره

$$S = 0 (D)$$

$$CV = \frac{S}{\bar{X}} * 100\%$$

Example In a class, if the mean is 30 and standard deviation is 2, find the Coefficient of variation for this class?

~~حل~~

$$CV = \frac{S}{\bar{X}} * 100\% = \frac{2}{30} * 100\% = 6.67\%$$

Example If the mean and Coefficient of variation of a data are 15 and 48 respectively, then find the value of stand deviation

$$\bar{X} = 15 \quad \left\{ \quad CV = \frac{S}{\bar{X}} \times 100$$

$$CV = 48 \quad \left\{ \quad 48 = \frac{S}{15} \times 100$$

$$S = 48 \times 0.15 = 7.2$$

Ex. 2
Example If $n = 5$, $\bar{X} = 6$, $\sum x^2 = 765$, then find the Coefficient of variation?

$$CV = \frac{S}{\bar{X}} \times 100\% \Rightarrow \frac{12.09}{6} \times 100\% = 201.55\%$$

$$S^2 = \frac{\sum x^2}{n-1} - \frac{(\sum x)^2}{n(n-1)}$$

$$= \frac{765}{5-1} - \frac{(30)^2}{5(4)}$$

$$= 146.25$$

$$\sum X = \bar{X} \cdot n$$

$$= 6 \times 5$$

$$= 30$$

$$S = \sqrt{146.25}$$

$$= 12.09$$

Example Find the Coefficient of Variation for (24, 26, 33, 37, 29, 31)?

$(x - \bar{x})$	-6	-4	3	7	-1
$(x - \bar{x})^2$	36	16	9	49	1

~~Q1~~ $CV = \frac{S}{\bar{X}} * 100\%$

$$\bar{X} = \frac{\sum X}{n} = \frac{180}{6} = 30$$

$$\sum (X - \bar{X})^2 = 112$$

$$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1} = \frac{112}{6 - 1} = 22.4$$

$$S = 4.73$$

$$CV = \frac{4.73}{30} * 100\% = 15.77\%$$

Example Two plants C and D of a factory show the following results about the number of workers and the wages paid to them.

no. of workers	5000 (C)	6000 (D)
Average monthly wages	2500	2500
Standard deviation	9	10

Using Coefficient of variation, find in which plant C or D, is there greater variability in individual wages

$$\cancel{CV} \quad CV_C = \frac{9}{2500} \times 100 = 0.36$$

$$CV_D = \frac{10}{2500} \times 100 = 0.40$$

$CV_D > CV_C$ \therefore D has greater variability

CV

SD

Relative measure
of variabilityabsolute measure
of variabilityused to compare
variability between
2 samplesused to measure
the dispersion of
data in a single
set

Example

The mean and SD of marks obtained by 40 students of a class in three subjects maths, science and social science are given

	Mean	SD
Maths	56	12
Science	65	14
Social Science	60	10

which of
three subjects
shows the highest
variation?

$$CV_{\text{maths}} = \frac{12}{56} \times 100\% = 21.43$$

$$CV_{\text{sci.}} = \frac{14}{65} \times 100\% = 21.54$$

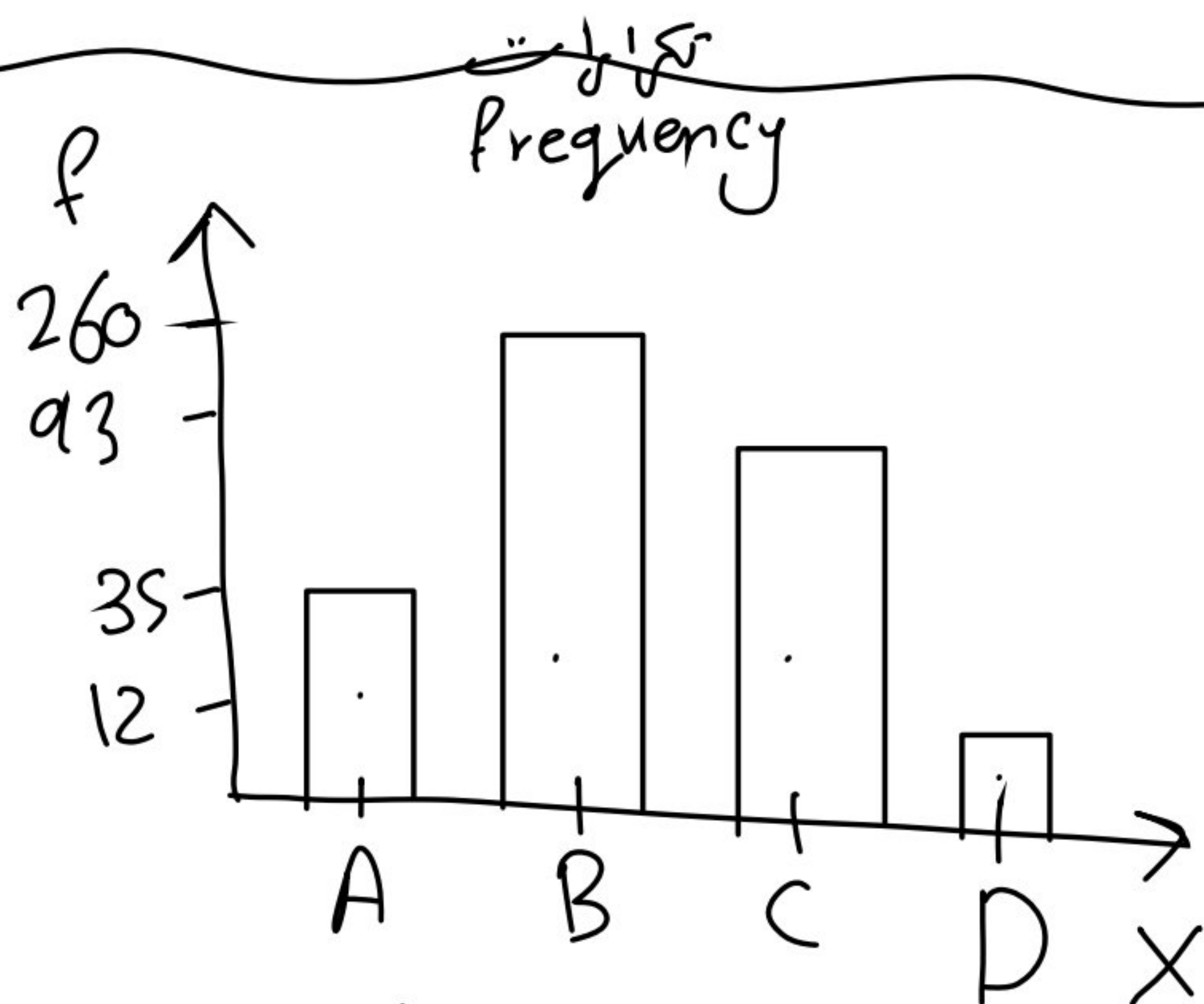
$$CV_{\text{Soc Sci.}} = \frac{10}{60} \times 100\% = 16.67$$

* Graphical methods

- ① Bar Graph
- ② Stem and leaf plot
- ③ Box plot

① Bar Graph

X	A	B	C	D
f	35	260	93	12



- difficult to construct

- identify sample points is lost

② Stem and leaf plot

الساق والأوراق

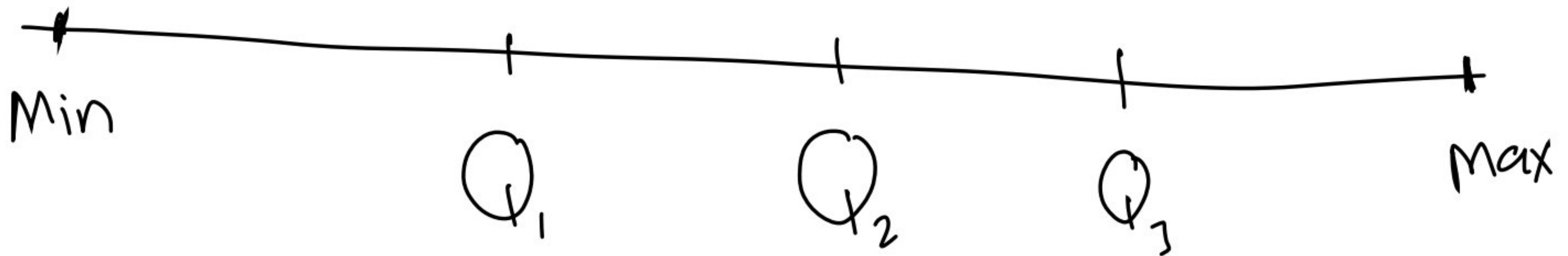
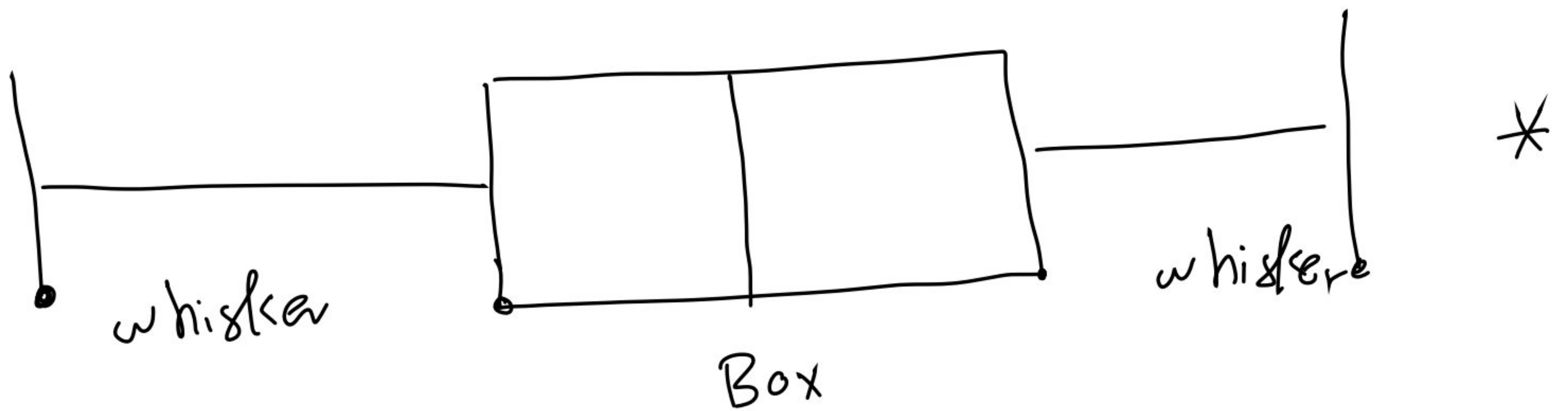
Example

23 71
58 71

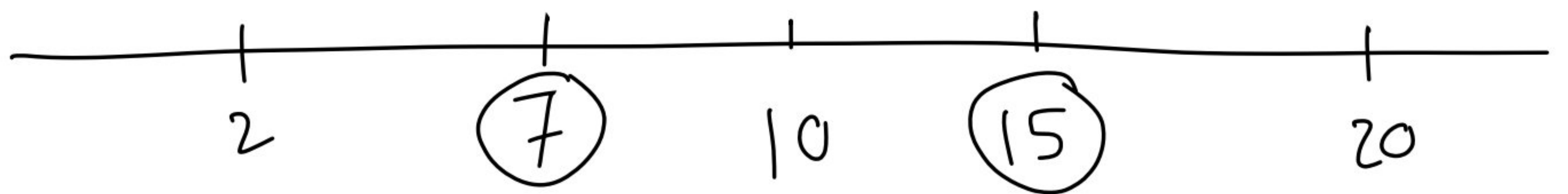
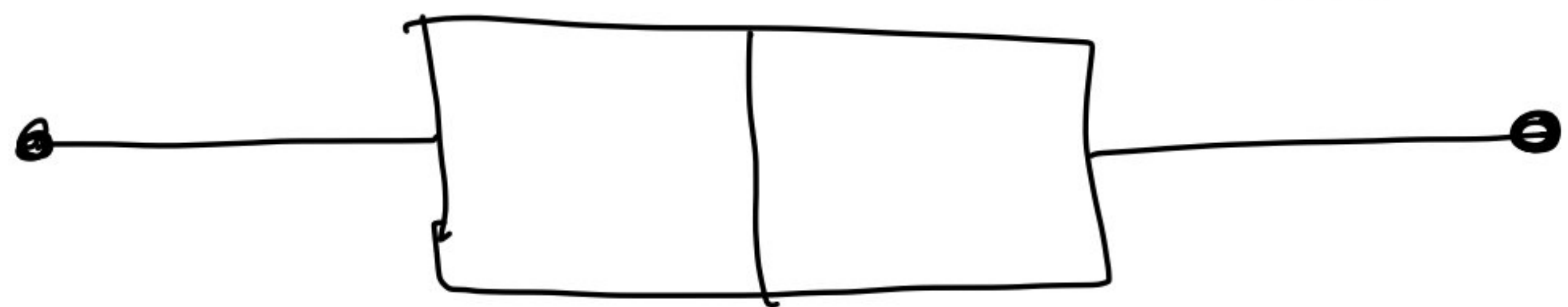
S	L
2	3
3	

62	72	4				
62	80	5	8			
63	82	6	2	2	3	5
65	82	7	1	1	2	
67	82	8	0	2	2	

3) Box Plot "Box and whiskers"



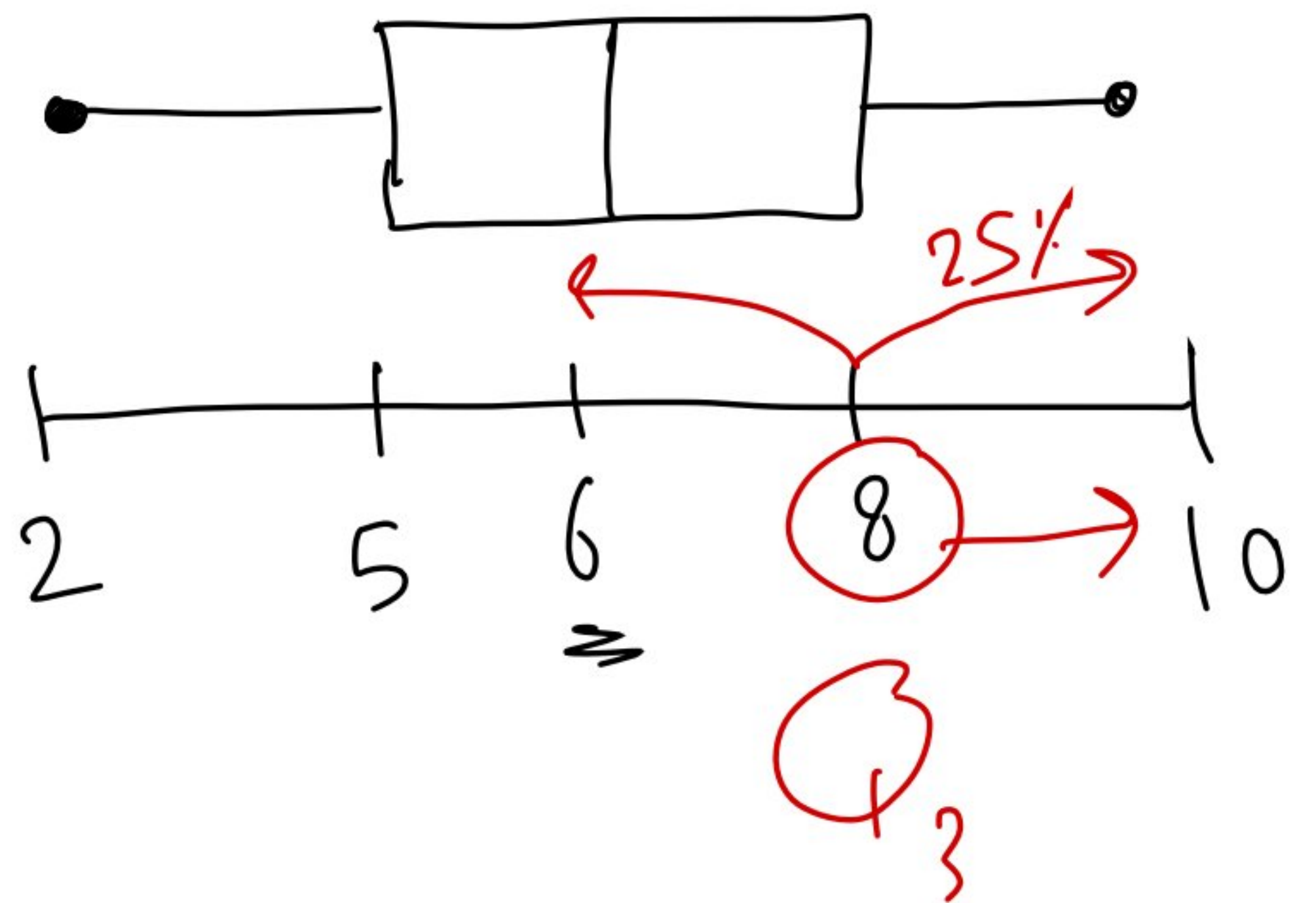
Example



find IQR ?

$$IQR = Q_3 - Q_1 = 15 - 7 = 8$$

Example If there are 32 students in the class, find:



① $Q_2 = 6$

② Range = $10 - 2 = 8$

③ $IQR = 8 - 5 = 3$

④ ^{9/10} The number of students achieved more than 8?

○. $25 * 32 = 8$

* Central tendency = Location

→ useful to define the center or middle of sample

→ Could be negative

→ Mean (generally not part of data set)

① oversensitive to extreme values

② easy to calculate

③ each sample has only one sample mean

→ Median (maybe part of data set)

⇒ ① less affected by outliers

② less efficient than mean

→ mode (always part of data set)

* Measures of variation (dispersion) (spread)

① Range

- Simplest MOV
- quick summary of variation
- extremely affected by outlier

② IQR

not affected by outlier

skewed to Right (+ skewness)

skewed to left (- skewness)

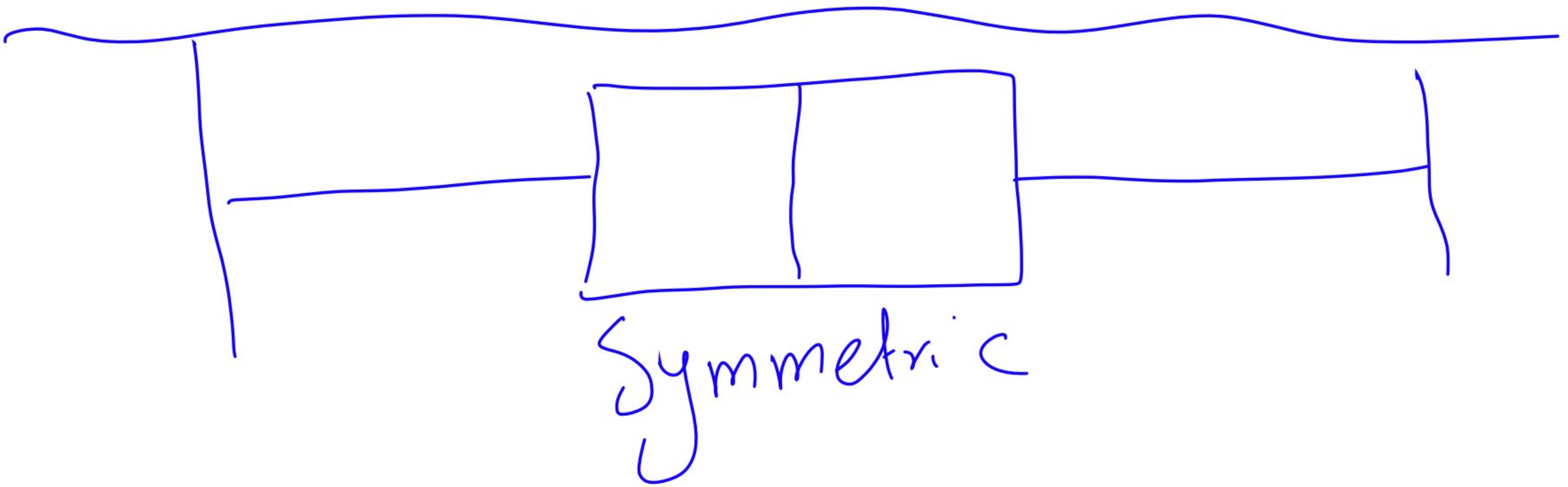
Variance

SD

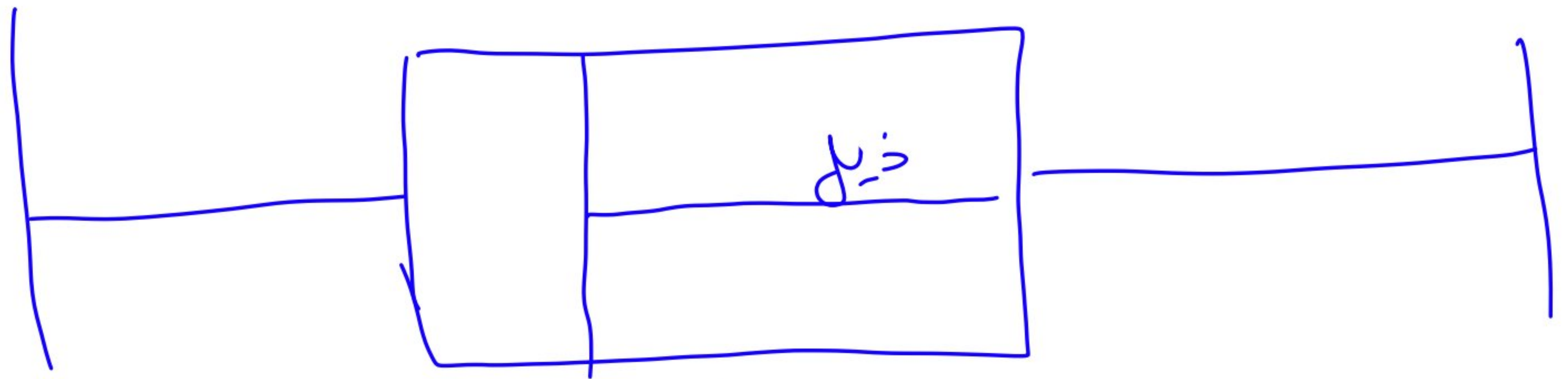
2, 2, 2, 2, 2

explains

$S^2 = 0$

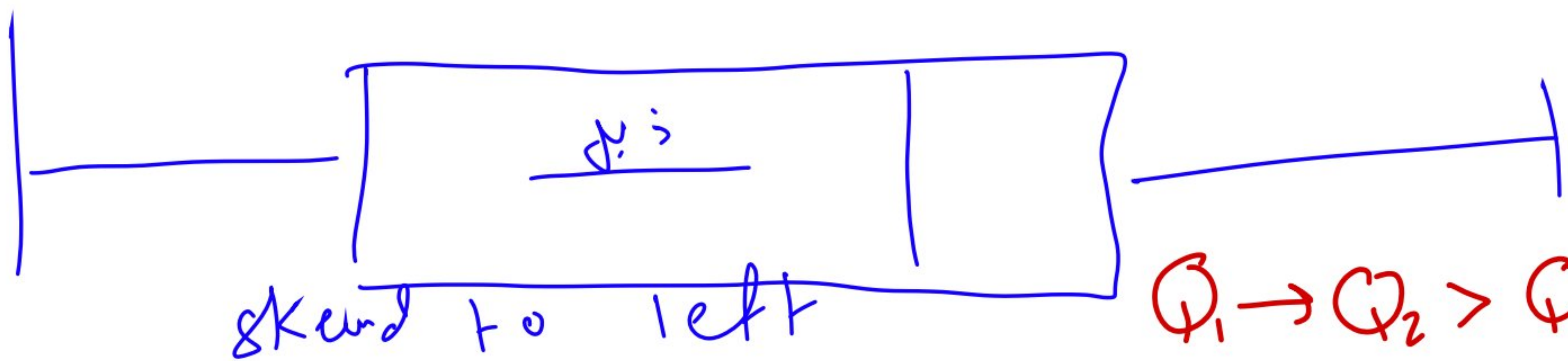


$Q_1 \rightarrow Q_2 = Q_2 \rightarrow Q_3$



skewed to Right

$Q_1 \rightarrow Q_2 < Q_2 \rightarrow Q_3$



skewed to left

$Q_1 \rightarrow Q_2 > Q_2 \rightarrow Q_3$

Chapter 3

Probability

علم الاحتمالات

تجارب

★ Experiment

① Deterministic مكددة

② Random عشوائية

★ The Sample space

(مجموعة النتائج)

Ω : defined as the set of all possible outcomes

مجموعة نتائج التجربة العشوائية

Example

Find

the sample space for

the following:

① Tossing ^{سحب} a fair Coin ^{عملة نقدية} 1-time

$$\Omega: \{H, T\} \quad 2^1$$

② Tossing a fair coin 2-times

$$\Omega: \left\{ \begin{array}{l} (H, H), (H, T) \\ (T, H), (T, T) \end{array} \right\} \quad 2^2$$

③ Tossing a fair coin 3-times

$$\Omega: \left\{ \begin{array}{l} (H, H, H), (H, H, T), (H, T, H), (H, T, T) \\ (T, H, H), (T, H, T), (T, T, H), (T, T, T) \end{array} \right\} \quad 2^3$$

NOTE when tossing a fair coin

k -times, so the number of elements

in a sample space is 2^k

1) Throwing a fair dice 1 -time
حجر النرد

$\Omega: \{1, 2, 3, 4, 5, 6\}$ 6^1

2) Throwing a fair dice 2 -times
 6^2

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

NOTE when throwing a fair dice

k times, so the number of elements in a sample space is 6^k

Example 2-cards are drawn from the Box that contained u -Cards numbered as $(1-u)$. Find the sample space when:

① The drawn was with replacement مع استرجاع

	1	2	3	u
1	$(1,1)$	$(1,2)$	$(1,3)$	$(1,u)$
2	$(2,1)$	$(2,2)$	$(2,3)$	$(2,u)$
3	$(3,1)$	$(3,2)$	$(3,3)$	$(3,u)$
u	$(u,1)$	$(u,2)$	$(u,3)$	(u,u)

1	2
3	u

② The drawn was without Replac. بدون استرجاع

	1	2	3	u
1	X	$(1,2)$	$(1,3)$	$(1,u)$
2	$(2,1)$	X	$(2,3)$	$(2,u)$
3	$(3,1)$	$(3,2)$	X	$(3,u)$
u	$(u,1)$	$(u,2)$	$(u,3)$	X

NOTE

☆ إذا السوال ما حدد مع

☆ الرجاء الرجوع يدونه الرجاء
نجد " يدونه الرجاء "

☆ The Event

☆ الحدث

☆ مجموعة من عناصر الفضاء
الاحتمالي

① Simple event: consist of 1 element
الحدث البسيط من عناصر الفضاء

② Composite "combined" event: consist of more
than 1 element of
sample space
الحدث المركب

③ Certain event: consist of all elements
of sample space
الحدث المؤكد

⊆ Impossible event : Consist of no elements of sample space
مستحيل

Example

Throwing a dice 1-time, define

$$\Omega: \{1, 2, 3, 4, 5, 6\}$$

A: {getting a number divisible by 5}

A: {5} → Simple event

B: {getting a prime number}

عدد أولي : يقبل القسمة على نفسه و على الواحد فقط
عدداً فقط (1)

B: {2, 3, 5} → Composite event

C: getting a number less than 7

C: {1, 2, 3, 4, 5, 6} → Certain event

D: getting a number more than 6

D: $\{ \} = \emptyset \rightarrow$ impossible event

* The probability of events

حتمال وقوع الأحداث

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{\text{عدد عناصر الحدث } A}{\text{عدد عناصر الفضاء العيني}}$$

Example In previous question:

$$\textcircled{1} P(A) = \frac{1}{6}$$

$$\textcircled{3} P(C) = \frac{6}{6} = 1$$

$$\textcircled{2} P(B) = \frac{3}{6} = \frac{1}{2}$$

$$\textcircled{4} P(D) = \frac{0}{6} = 0$$

NOTES

~~24P~~

① certain events probability = 1

② impossible events probability = 0

③ $P(\Omega) = 1$

④ $P(\emptyset) = 0$

24P

⑤ $0 \leq P(A) \leq 1$

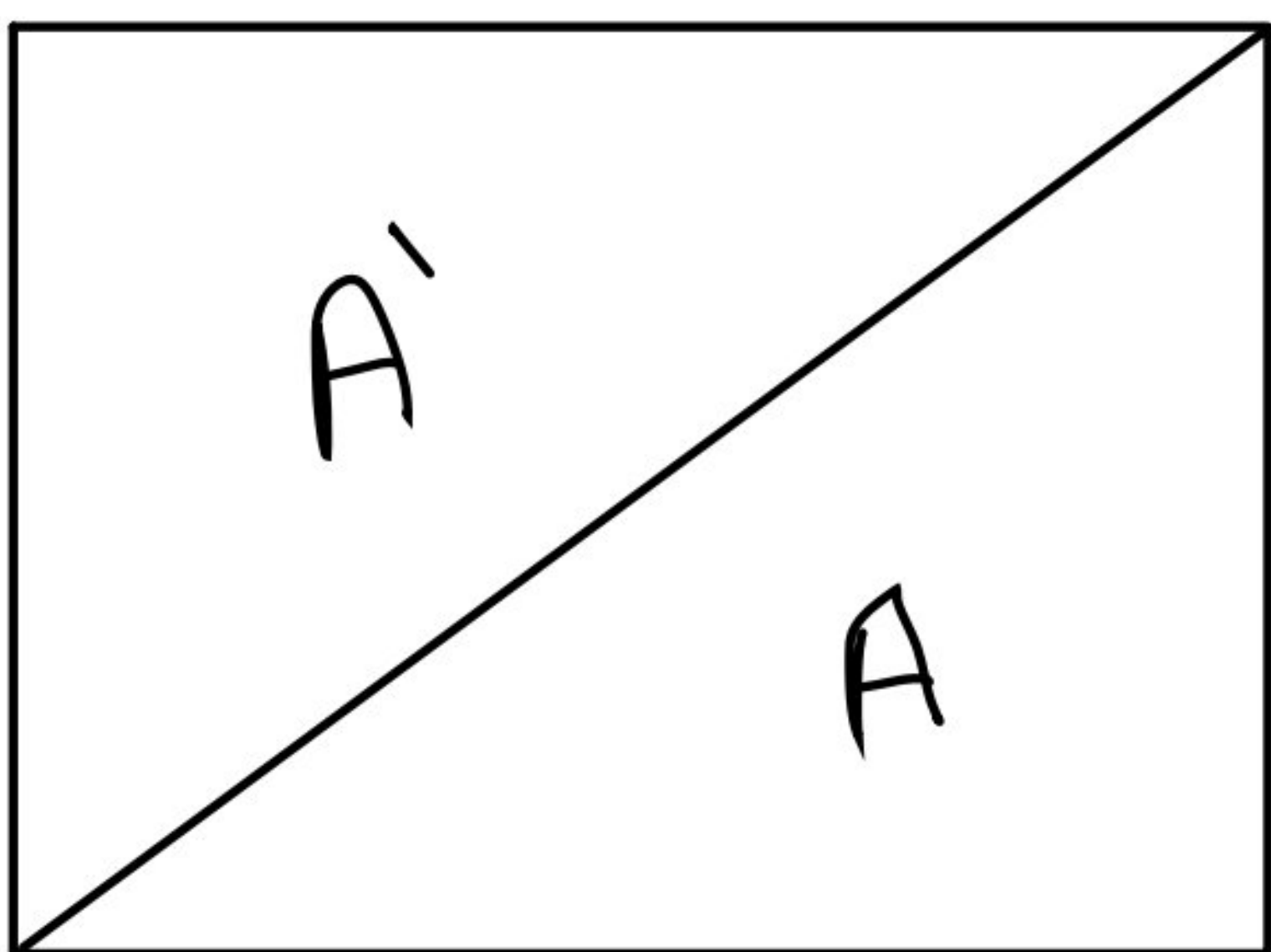
Rules of probability

A: الاحتمال الاول

B: الاحتمال الثاني

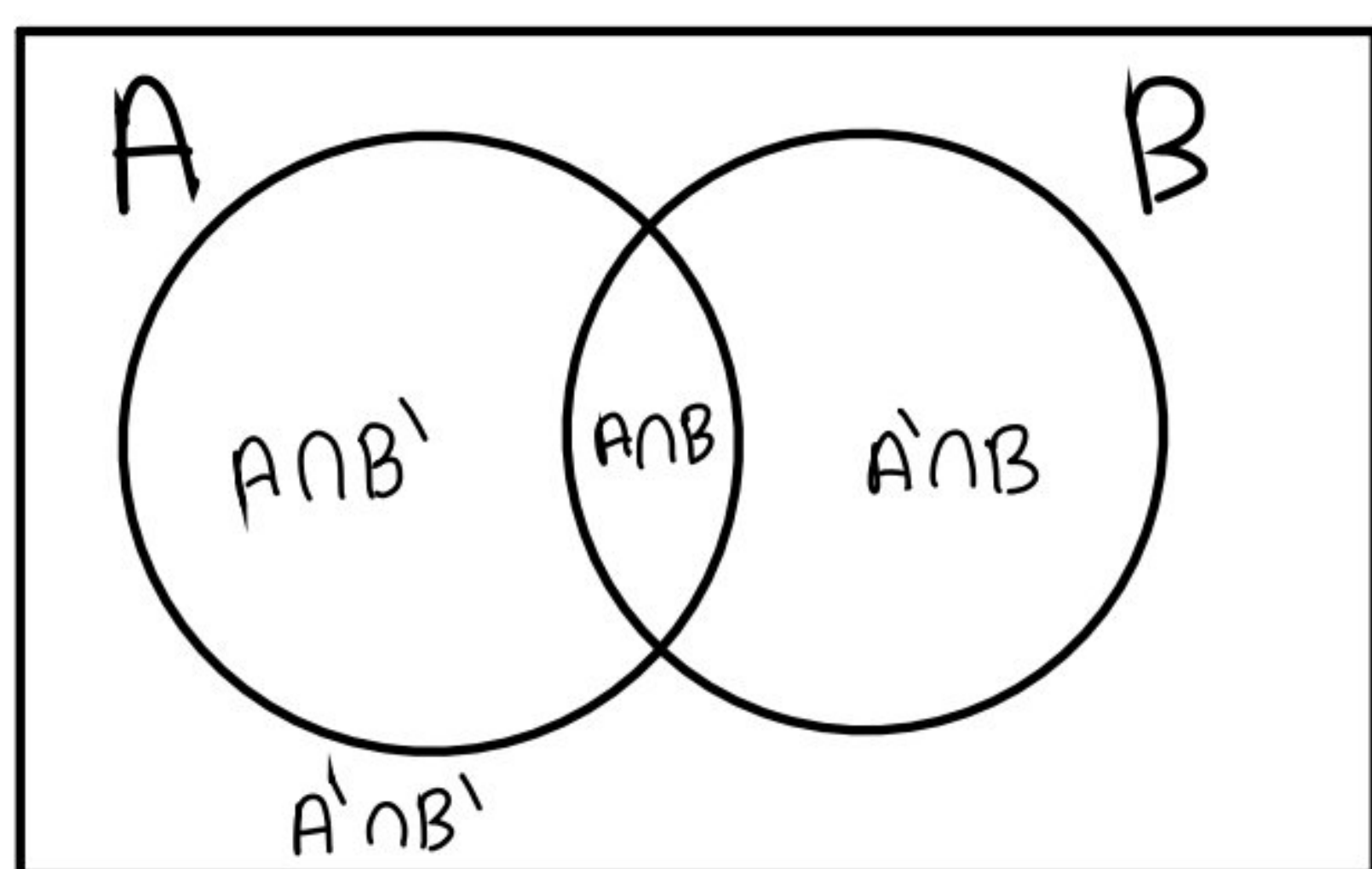
قوانين الاحتمال

intersection \cap تقاطع
union \cup اتحاد



$$P(A) + P(A') = 1$$

① $P(A') = 1 - P(A)$



② $P(A \cap B') = P(A) - P(A \cap B)$
 $P(A' \cap B) = P(B) - P(A \cap B)$

$$\textcircled{3} P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

"Addition Rule"

$$\textcircled{4} P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$$

"De Morgan's Law"

$\textcircled{5}$ A and B are mutually exclusive
if : $\textcircled{1} P(A \cap B) = 0$ "disjoint"

and $\textcircled{2} P(A \cup B) = P(A) + P(B)$

$\textcircled{6}$ A and B are independent if
 $P(A \cap B) = P(A) \cdot P(B)$

NOTE A & B & C independent

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

"multiplication Rule"

$$\textcircled{7} P(A|B) = \frac{P(A \cap B)}{P(B)}$$

given if

"Conditional probability"

NOTES

① $P(A|B)$ and A & B are independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$P(A|B) \equiv P(A)$$

$$P(A|B') = P(A)$$

$$\textcircled{2} P(A \cap B) \equiv P(B \cap A) \quad A \text{ and } B$$

$$\textcircled{3} P(A \cup B) \equiv P(B \cup A) \quad A \text{ or } B$$

Example If $P(A) = 0.8$, $P(B) = 0.7$
 $P(A \cap B) = 0.6$ find:

$$i) P(A') = 1 - P(A) = 1 - 0.8 = 0.2$$

$$ii) P(B') = 1 - P(B) = 1 - 0.7 = 0.3$$

$$iii) P(A \cap B') = P(A) - P(A \cap B) \\ = 0.8 - 0.6 = 0.2$$

$$iv) P(A' \cap B) = P(B) - P(A \cap B) \\ = 0.7 - 0.6 = 0.1$$

$$v) P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0.8 + 0.7 - 0.6 = 0.9$$

$$vi) P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) \\ = 1 - 0.9 \\ = 0.1$$

$$vii) P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) \\ = 1 - 0.6 = 0.4$$

حلوة
viii) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
 $= 0.2 + 0.7 - 0.1 = 0.8$

ix) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$
 $= 0.8 + 0.3 - 0.2 = 0.9$

x) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.7} = \frac{6}{7}$

xi) $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.1}{0.7} = \frac{1}{7}$

xii) $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.2}{0.3} = \frac{2}{3}$

xiii) $P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{0.1}{0.3} = \frac{1}{3}$

xiv) $P(A|B)' = 1 - P(A|B) = 1 - \frac{6}{7}$
 $= \frac{1}{7}$

NOTE $P(A|B)' = P(A'|B)$

Example If $P(A) = 0.6$, $P(B) = 0.5$ and

$P(A \cup B) = 0.8$ are A & B mutually exclusive?
independent? neither?

Mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

$$0.8 \stackrel{?}{=} 0.6 + 0.5$$

$$0.8 \stackrel{?}{=} 1.1$$

(Not ME)

independent

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.5 + 0.6 - P(A \cap B)$$

$$0.8 = 1.1 - P(A \cap B)$$

$$P(A \cap B) = 1.1 - 0.8$$

$$P(A \cap B) = 0.3$$

$$0.3 \stackrel{?}{=} 0.5 * 0.6$$

$$0.3 = 0.3 \quad \checkmark$$

So A & B are independent events

Example If A and B are independent events such that $P(A) = 2 * P(B)$ and $P(A \cup B) = 0.8$ then find $P(A)$?

sol $P(A \cap B) = P(A) \cdot P(B)$

$$P(A) = 2 * P(B)$$

$$P(A \cup B) = 0.8$$

$$0.8 = P(A) + P(B) - P(A \cap B)$$

$$0.8 = P(A) + P(B) - P(A) \cdot P(B)$$

$$0.8 = 2 * P(B) + P(B) - 2P(B) * P(B)$$

$$X = P(B)$$

$$0.8 = 2X + X - 2X^2$$

$$2X^2 - 3X + 0.8 = 0$$

$$X = 1.53$$

X

$$X = 0.34$$

✓

$$P(B) = 0.34$$

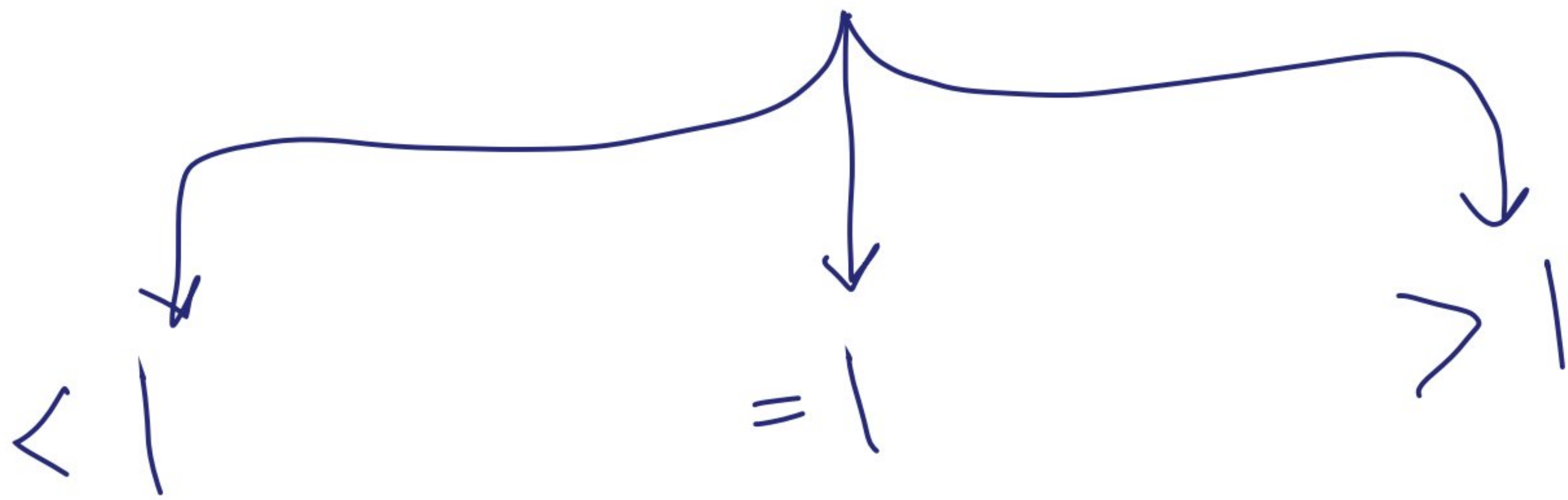
$$P(A) = 2 * 0.34 = 0.68$$

* Relative Risk (RR)

Risk factor
ریسک فاکتور

"Risk ratio"

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{\text{Risk for exposure group}}{\text{Risk for unexposure group}}$$



* Some interpretations

$\Rightarrow RR = 1.5$

exposure group risk is higher than unexposure groupe in about 0.50

$\Rightarrow RR = 3$

exposure group risk is higher than unexposure groupe in about 3 times

$\Rightarrow RR = 0.8$

exposure group risk is less than unexposure groupe in about 0.20

NOTE IF A and B are independent
then the RR is 1

~~زیر~~

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{\cancel{P(B)}}{\cancel{P(B)}} = 1$$

Example A study enrolls a 100 smokers and 100 non-smokers. They are followed for next years for developing ^{درد} lung CA. 30 of smokers and 10 of non-smokers developed lung CA. Calculate the RR?

~~زیر~~

	<u>Smokers</u>	<u>non-smokers</u>
	100	100
CA	30	10

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{30/100}{10/100} = 3$$

Example If 1 in 10 people exposed to a substance gets sick. If 1 in 100 people who are not exposed get sick. Find the RR?

	Exposed	Not Exposed
Sick	1	1

$$RR = \frac{1/10}{1/100} = 10$$

Example Suppose we want to know if exercise affects the risk of developing some disease we collect data and find that 28% of people who exercise regularly develop this disease while 50% of people who do not exercise

regularly develop this disease, find the RR?

~~31~~ $RR = \frac{0.28}{0.50} = 0.56$

Example

Suppose we want to know if

Some new studying program affects the ability of students to pass a particular exam. we collect a data and find that 40% of students who use the new studying program pass the exam while 40% of students who do not use the studying program also pass the exam, calculate the RR?

~~31~~ $RR = \frac{0.40}{0.40} = 1$

Example

Suppose 50 basketball players

use a new training program, and 50 players

use an old training program. At the end of the program we test each player to see if they pass a certain skills test. find RR?

	passed	failed	
New program	34	16	= 50
old program	39	11	= 50

~~(1)~~
(1) $RR = \frac{34/50}{39/50} = 0.872$

(2) $RR = \frac{P(B|A)}{P(B|A')} = \frac{\frac{P(B \cap A)}{P(A)}}{\frac{P(B \cap A')}{P(A')}} = \frac{\frac{34}{34+16}}{\frac{39}{39+11}} = \frac{34/50}{39/50} = 0.872$

B: pass
A: new program

Example Suppose that among 100,000 women with negative mammograms 20 will be diagnosed with breast CA within years, whereas 1 woman in 10 with positive mammograms will be diagnosed by breast CA within years
 find RR?

$$P(B/A') = 0.002$$

$$P(B/A) = 0.1$$

	(+)	(-)
B	10	100,000
Breast CA	1	20

$$RR = \frac{1/10}{20/100,000} = 500$$



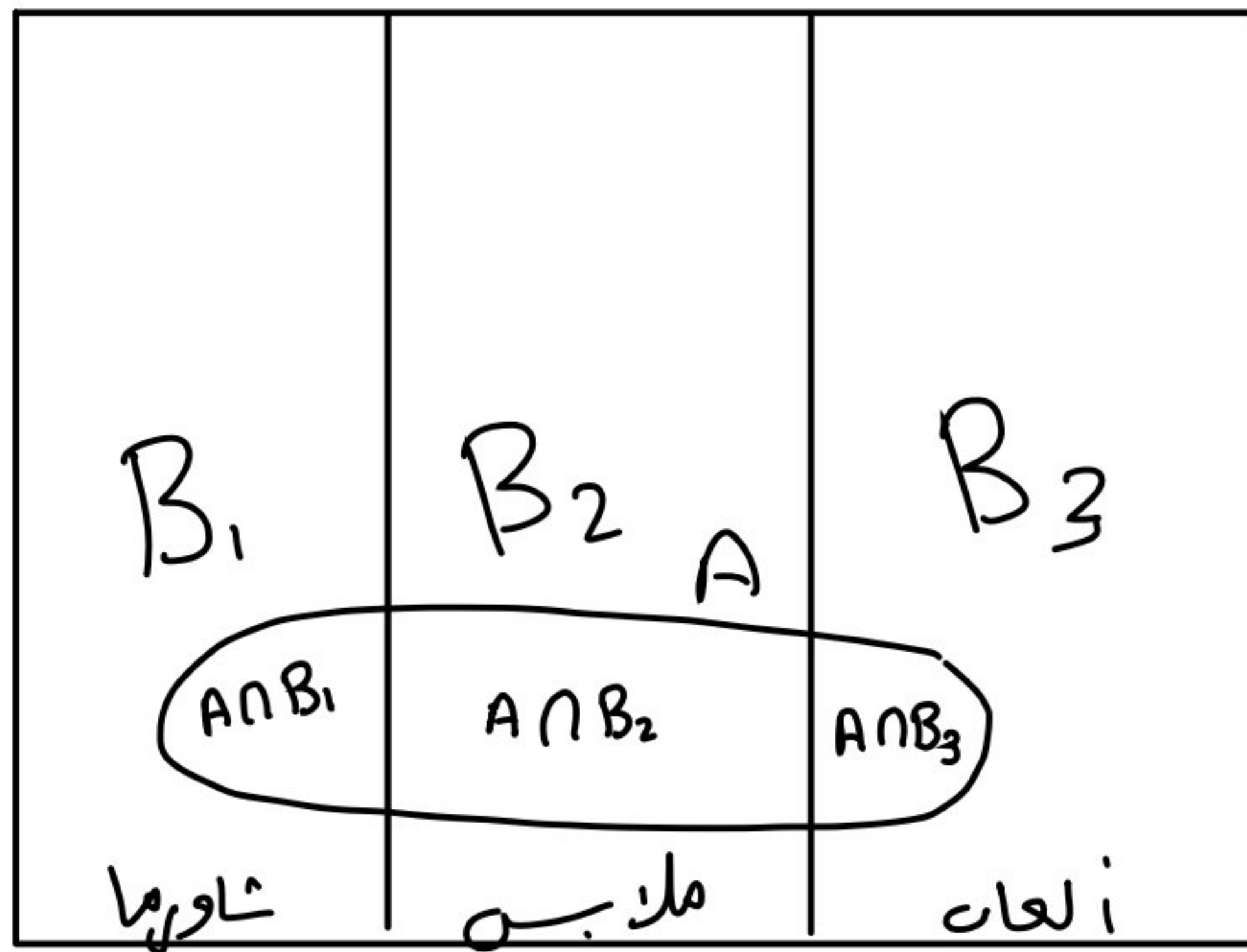
* Total probability

$$\Omega = 1$$

* mutually exclusive events

$A \& B$

$\Rightarrow \bigcup A \Rightarrow$ قسم
حدث
B



$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$$

$$\frac{P(A|B_i)}{1} = \frac{P(A \cap B_i)}{P(B_i)} \Rightarrow P(A \cap B_i) = P(A|B_i) * P(B_i)$$

$$P(A) = P(A|B_1) * P(B_1) + P(A|B_2) * P(B_2) + P(A|B_3) * P(B_3)$$

$$P(A) = \sum P(A|B_n) * P(B_n)$$

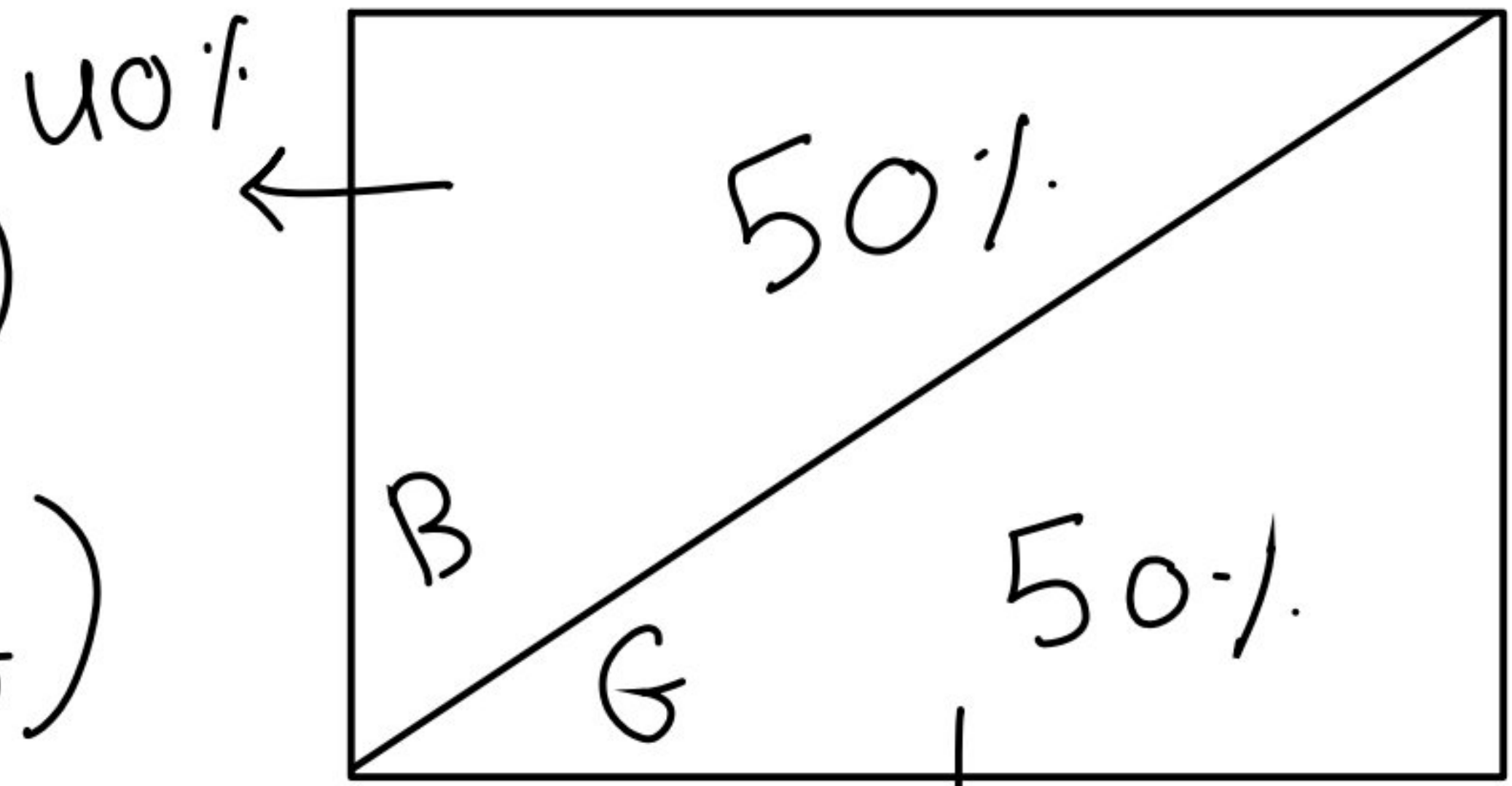
\Rightarrow insurance

\Rightarrow disease



Example If 40% of Boys opted for maths and 60% of Girls opted for maths, then what is the probability that math is chosen if half of class's population is girls?

$$P(\text{maths}) = P(\text{maths} \cap B) + P(\text{maths} \cap G)$$



$$= P(\text{math} | B) * P(B) + P(\text{math} | G) * P(G)$$

$$= 0.40 * 0.50 + 0.60 * 0.50$$

$$= 0.50$$

Example Company A produces 10% defective products, Company B produces 20% defective products and Company C produces 5% defective products

products. If choosing a company is an equally likely event, find the probability that the product chosen is defective?

~~b1~~

A	B	C
10% D	20% D	5% D
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$P(\text{defective}) =$$

$$P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$= P(D|A) * P(A) + P(D|B) * P(B) + P(D|C) * P(C)$$

$$= 0.10 * \frac{1}{3} + 0.20 * \frac{1}{3} + 0.05 * \frac{1}{3}$$

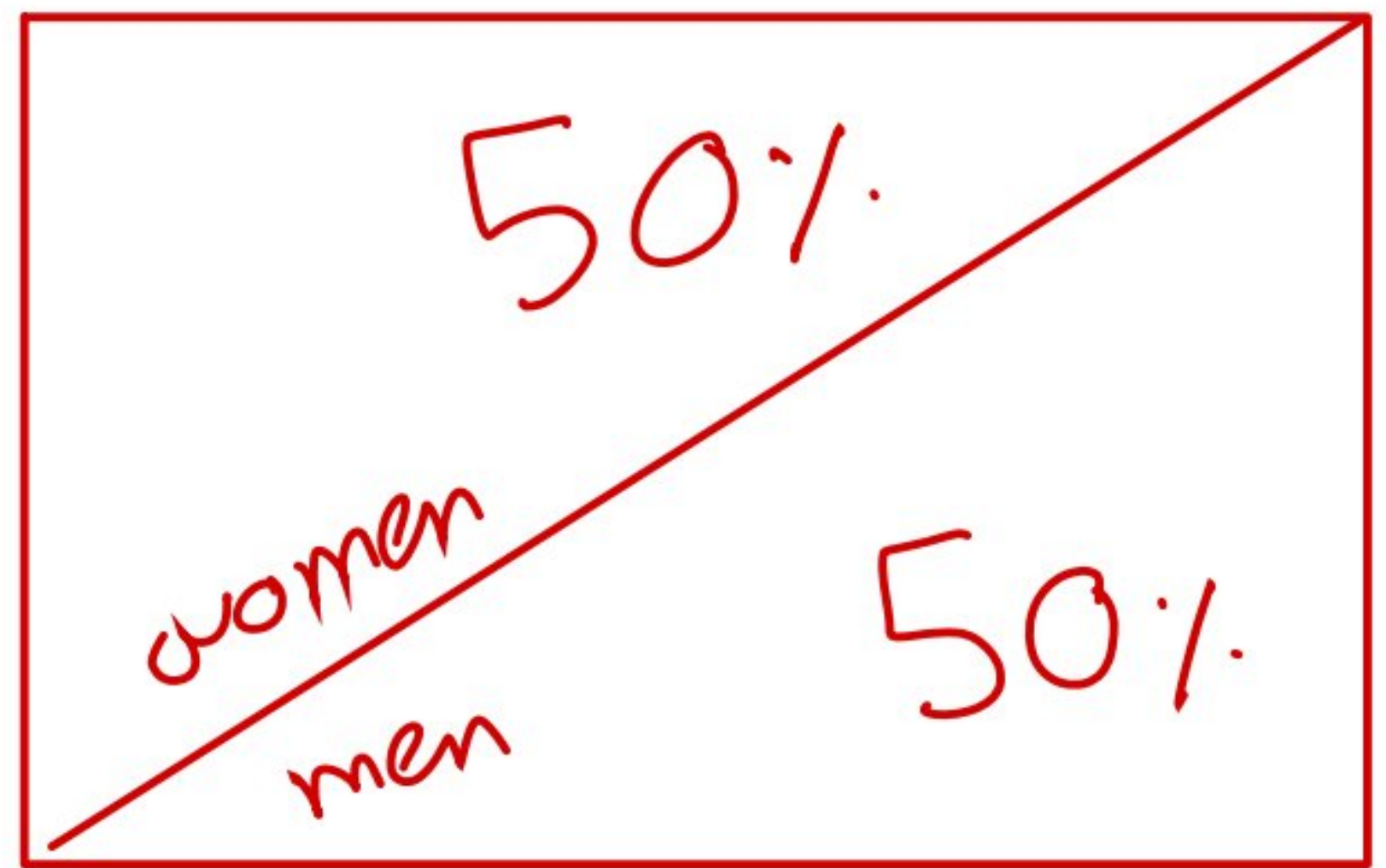
$$= 0.12$$

Example Suppose 5 men out of 100 and 10 women out of 250 are Color Blind, then find the total probability of Color blind people?

(Assume that both men and women are equally in number)

$$P(CB) = P(CB \cap w) + P(CB \cap m)$$

$$\frac{10}{250}$$



5%

$$= P(CB|w) * P(w) + P(CB|m) * P(m)$$

$$= \frac{10}{250} * 0.50 + \frac{5}{100} * 0.50$$

$$= 0.045$$

Example

we are
المربي البصيا/الزرقا

∴ planning a 5 year

study of Cataract in a population of 5000 people

60 years old and older. we know that:

$A_1: \{\text{ages } 60-64\}$ $A_2: \{\text{Ages } 65-69\}$

$A_3: \{ \text{ages } 70-74 \}$ $A_4: \{ \text{Ages } 75+ \}$

what is the probability of event B which is defined as the probability of developing cataract in the next 5 years, given:

$$P(A_1) = 0.45$$

$$P(A_2) = 0.28$$

$$P(A_3) = 0.20$$

$$P(A_4) = 0.07$$

$$P(B|A_1) = 0.024$$

$$P(B|A_2) = 0.046$$

$$P(B|A_3) = 0.088$$

$$P(B|A_4) = 0.153$$

~~Q21~~

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3) + P(B \cap A_4)$$

$$P(B) = P(B|A_1) * P(A_1) + P(B|A_2) * P(A_2) + P(B|A_3) * P(A_3) + P(B|A_4) * P(A_4)$$

$$= 0.024 * 0.45 + 0.046 * 0.28 + 0.088 * 0.2 + 0.153 * 0.07$$

$$= 0.05$$

Example Suppose 2 doctors A & B, test all patients coming into clinic for syphilis. Let us define the following 2 events:

A^+ (doctor A makes a positive diagnosis)

B^+ (doctor B makes a positive diagnosis)

$$P(A^+) = 0.10, P(B^+) = 0.17, P(A^+ \cap B^+) = 0.08$$

Answer the following:

a) find the conditional probability that doctor B makes a positive diagnosis given that doctor A makes a positive diagnosis?

$$P(B^+ | A^+) = \frac{P(B^+ \cap A^+)}{P(A^+)} = \frac{0.08}{0.10} = 0.8$$

B) what is the conditional probability that doctor B makes a positive diagnosis given that doctor A makes a negative diagnosis?

$$P(B+ | A^+) = \frac{P(B+ \cap A^+)}{P(A^+)} = \frac{0.17 - 0.08}{0.90} = 0.1$$

$$P(B+ \cap A^+) = P(B+) - P(A^+ \cap B+)$$

c) what is the RR of B+ given A+?

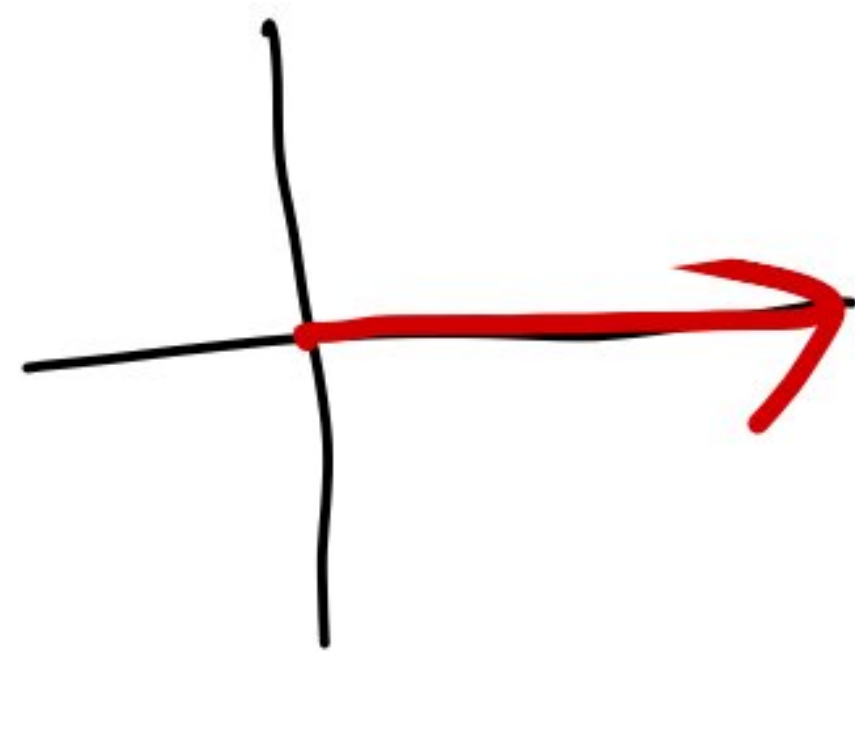
$$RR = \frac{P(B+ | A^+)}{P(B+ | A^+)} = \frac{0.8}{0.1} = 8$$

★ Baye's Rule and Screening tests
الحقيقة

	(+)	(-)
(+)	True positive	false positive
(-)	false negative	True negative

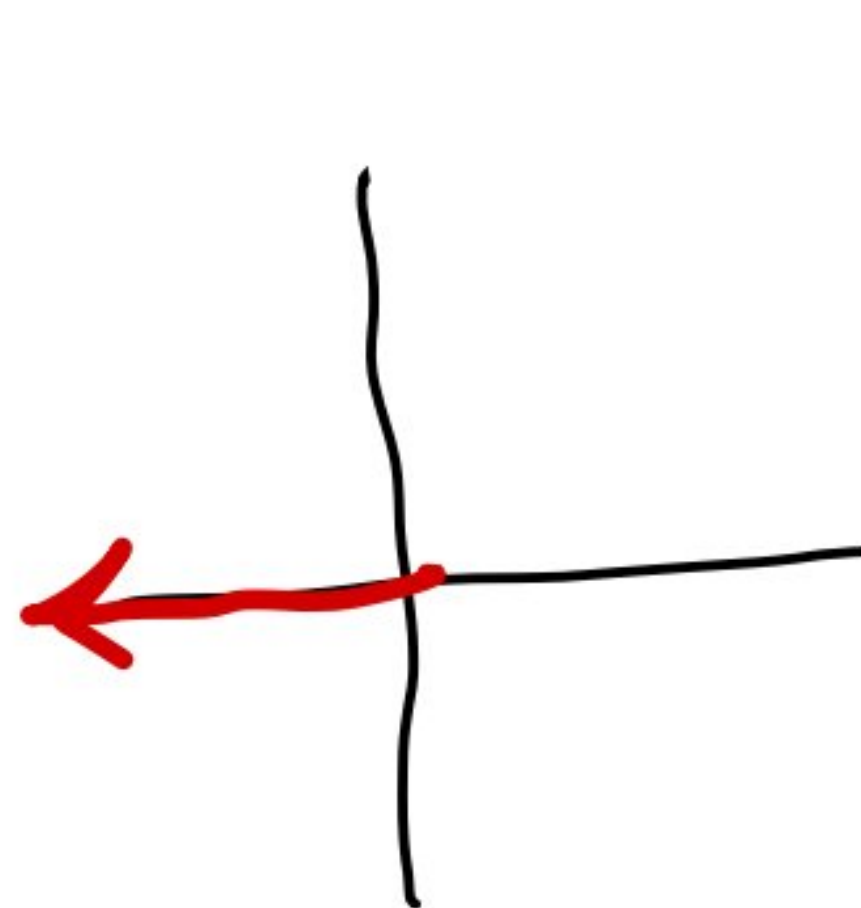
① positive predictive value

$$PPV \equiv PV+ = \frac{TP}{TP+FP}$$

 = $P(\text{disease} | +)$

② negative predictive value

$$NPV \equiv PV(-) = \frac{TN}{TN+FN}$$

 = $P(\text{no disease} | (-))$

③ Sensitivity = $\frac{TP}{FN+TP}$

$$= P(+ | \text{disease})$$

④ Specificity = $\frac{TN}{TN+FP}$

$P(-) \mid \text{no disease}$

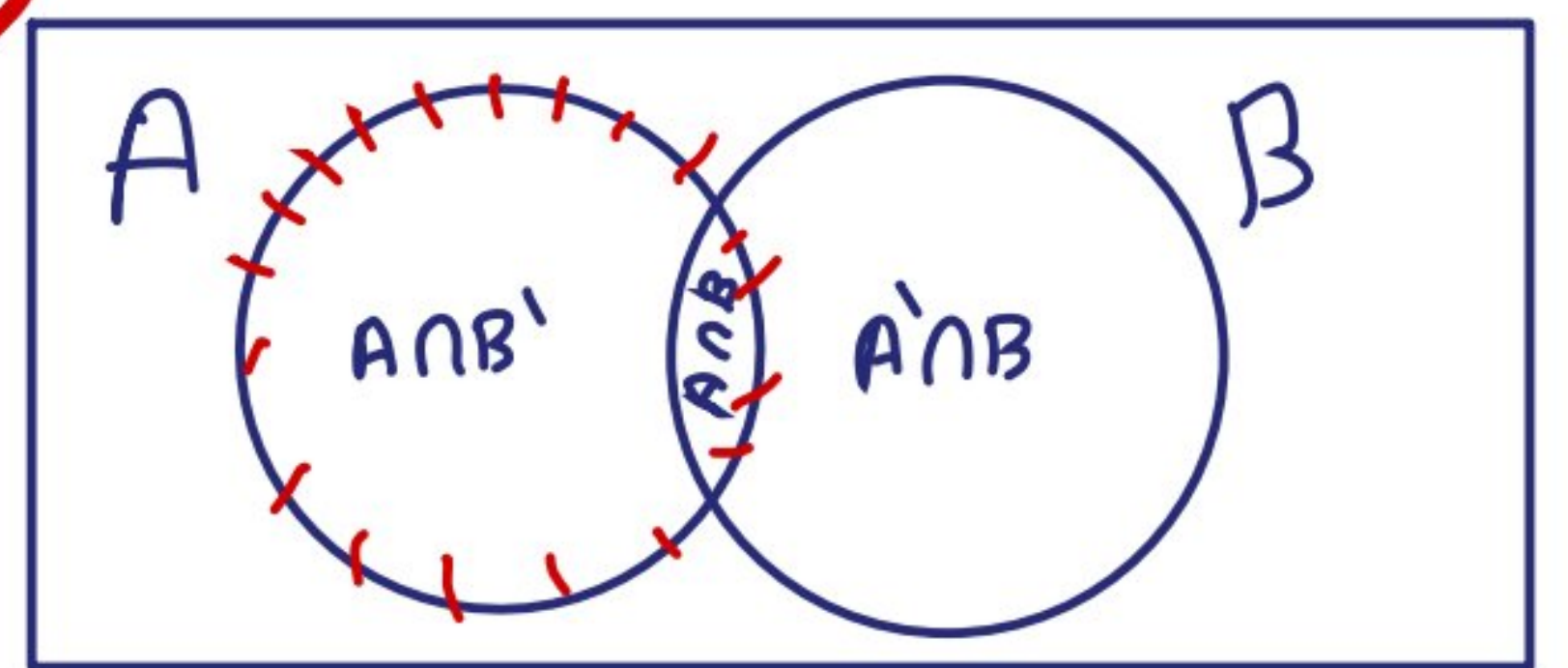
* Baye's Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$P(B \cap A) = P(A \cap B)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B) * P(B)}{P(A \cap B) + P(A \cap B')}$$

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$



Example

The table below shows the results

from looking at the diagnostic accuracy of a new rapid test for HIV in 100,000

Subjects, The rows of the table represent the test result and the columns the true disease status

	HIV(+)	HIV(-)	Total
Test (+)	378	397	775
Test (-)	2	98823	98825
Total	380	99220	100000

① find $P_{V(+)}$ (PPV)?

$$PPV = P(\text{disease} | (+)) = \frac{P(\text{disease} \cap (+))}{P(+)}$$
$$= \frac{378}{775}$$

② find NPV ($P_{V(-)}$)?

$$NPV = P(\text{no disease} | (-)) = \frac{P(\text{no} \cap -)}{P(-)}$$

$$= \frac{98823}{98825}$$

③ find sensitivity?

$$P(+ | \text{disease}) = \frac{P(+ \cap \text{disease})}{P(\text{disease})} = \frac{378}{380}$$

④ find the specificity?

$$P(- | \text{no disease}) = \frac{P(- \cap \text{no})}{P(\text{no})} = \frac{98823}{99220}$$

Example In a hospital, If you know that probability to have certain disease is 0.15 and the probability to have a positive

test if you have the disease is 0.95
 and a negative test if you don't have
 the disease is 0.93, and a positive
 test if you don't have the disease is 0.7

find $P(V(+))$?

$$\begin{aligned} P(B') &= 1 - P(B) \\ &= 1 - 0.15 \\ &= 0.85 \end{aligned}$$

~~B~~

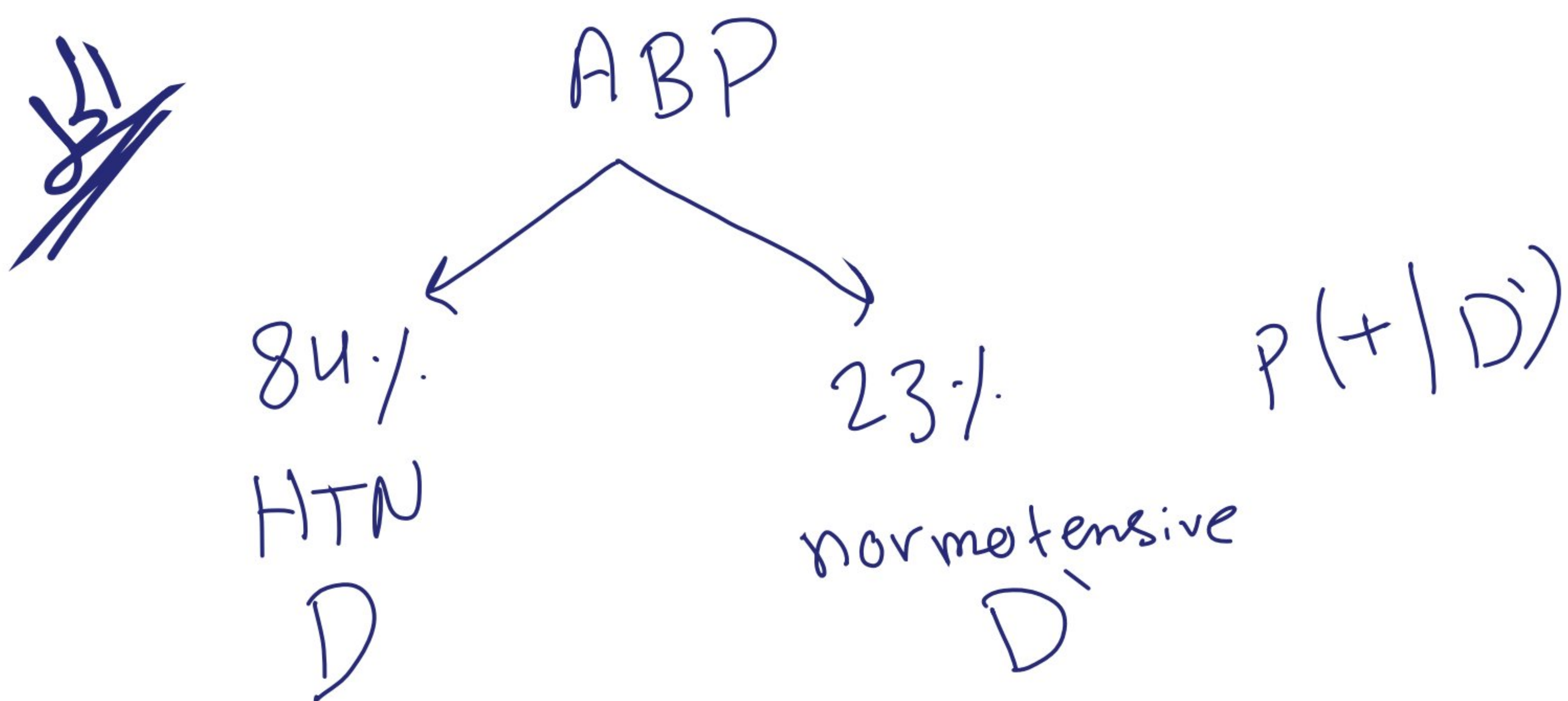
$P(A B)$	$P(D) = 0.15$	
$P(A' B')$	$P(+ D) = 0.95$	→ sensitivity
$P(A B')$	$P(- D') = 0.93$	→ specificity
	$P(+ D') = 0.7$	→ false positive

$$P(V(+)) = P(D|+)$$

A: (+)
 B: disease

$$\begin{aligned} P(B|A) &= \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')} \\ &= \frac{0.95 * 0.15}{0.95 * 0.15 + 0.7 * 0.85} \end{aligned}$$

Example Suppose 84% of hypertensive and 23% of normotensive are classified by Automated blood pressure machine as hypertensive what are the (P_{V+}) and (P_{V-}) of the machine if you know that 20% of the adult population have the disease?



$$P\left(\overset{A}{+} \mid \overset{B}{D}\right) = 0.84 \rightarrow \text{sensitivity}$$

$$P\left(\overset{A}{+} \mid \overset{B'}{D'}\right) = 0.23$$

A: (+)

B: D

$$P(A' | B') = 1 - P(A | B') = 1 - 0.23 = \boxed{0.77}$$

$$P\left(\underset{B}{D}\right) = 0.20$$

NOTE = $P(A' | B') = 1 - P(A | B')$

$$\begin{aligned}
 P_{V(+)} = P(B|A) &= \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')} \\
 &= \frac{0.84 * 0.20}{0.84 * 0.20 + 0.23 * 0.80} \\
 &= 0.48
 \end{aligned}$$

$$\begin{aligned}
 P_{V(-)} = P(B'|A') &= \frac{P(A'|B') * P(B')}{P(A'|B') * P(B') + P(A'|B) * P(B)} \\
 &= \frac{0.77 * 0.80}{0.77 * 0.80 + 0.16 * 0.20} \\
 &= 0.95
 \end{aligned}$$

NOTE $P(A'|B) = P(A|B)' = 1 - P(A|B)$

Example

A man is known to speak the

truth 2 out of 3 times. He throws a dice and reports that the number obtained

is 4. Find probability that actually the number obtained is 4?

$$P\left(\begin{matrix} A \\ \text{50} \\ 4 \end{matrix} \middle| \begin{matrix} B \\ 4 \end{matrix}\right) = \frac{2}{3}$$

$$\Omega: \{1, 2, 3, 4, 5, 6\}$$

$$P(4) = \frac{1}{6}$$

$$P\left(\begin{matrix} A \\ \text{50} \\ 4 \end{matrix} \middle| \begin{matrix} B' \\ 4' \end{matrix}\right) = \frac{1}{3}$$

$$P(4) = \frac{1}{6}$$

$$P\left(\begin{matrix} B \\ 4 \end{matrix} \middle| \begin{matrix} A \\ \text{50} \\ 4 \end{matrix}\right) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{\frac{2}{3} * \frac{1}{6}}{\frac{2}{3} * \frac{1}{6} + \frac{1}{3} * \frac{5}{6}} = \frac{2}{7}$$

$$\frac{2}{3} * \frac{1}{6} + \frac{1}{3} * \frac{5}{6}$$

Example

suppose 50 men out of 1000 men

and 25 women out of 2000 are orators

An orator is chosen at random. Find the probability that a male person is selected?

$$P(\overset{\textcircled{A}}{\text{orator}} | \overset{\textcircled{B}}{\text{man}}) = \frac{50}{1000}$$

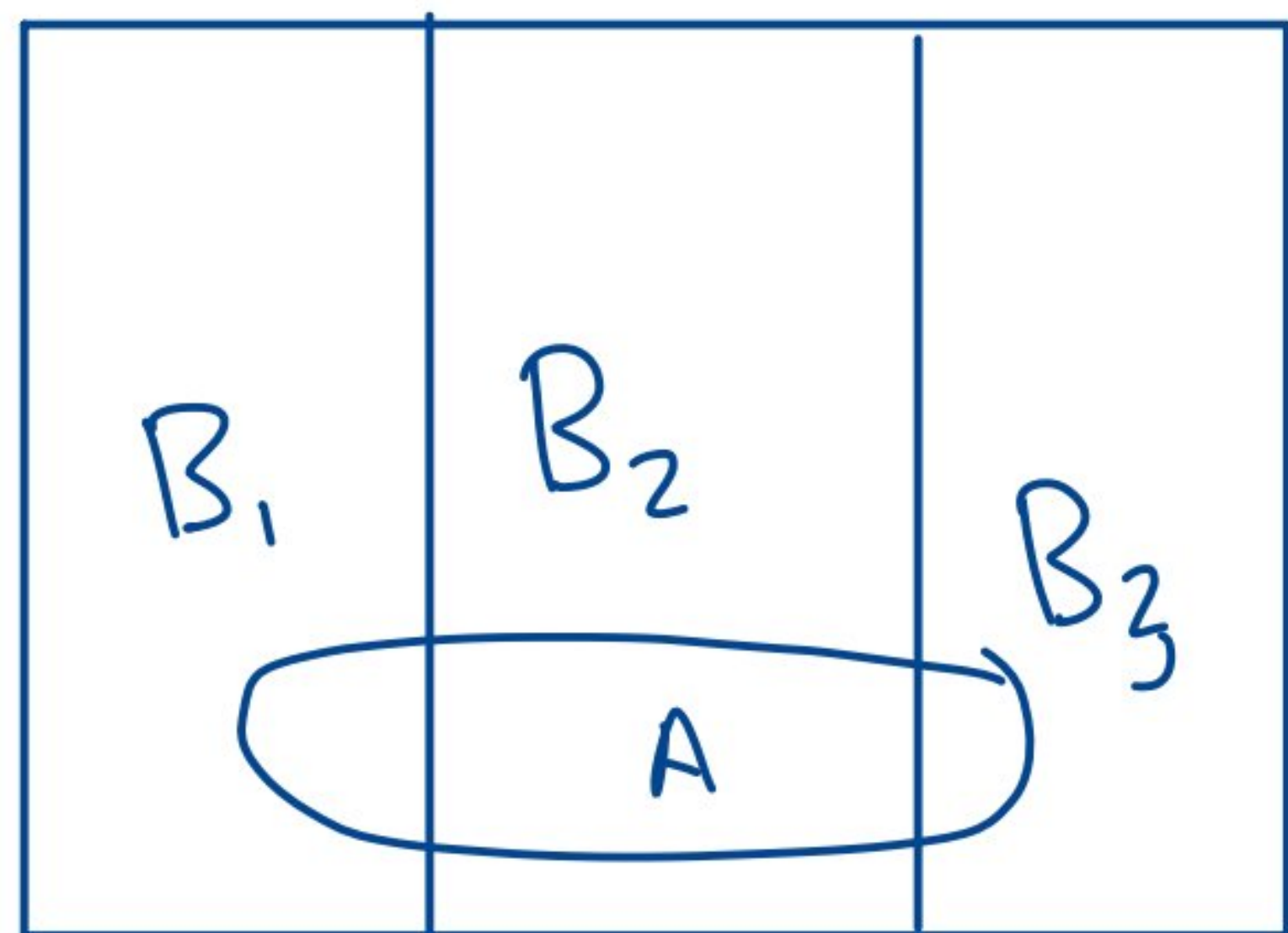
$$P(\overset{\textcircled{A}}{\text{orator}} | \overset{\textcircled{B'}}{\text{women}}) = \frac{25}{1000}$$

$$P(\overset{B}{\text{man}} | \overset{A}{\text{orator}}) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{\frac{50}{1000} * \frac{1}{2}}{\frac{50}{1000} * \frac{1}{2} + \frac{25}{1000} * \frac{1}{2}} = \text{---}$$

* Generalized Baye's Rule

$$P(A) = \sum P(A|B_n) * P(B_n)$$



$$P(B_n|A) = \frac{P(A|B_n) * P(B_n)}{P(A)} = \frac{P(A|B_n) * P(B_n)}{\sum P(A|B_n) * P(B_n)}$$

Example Three factories A, B and C of an electrical bulb produce respectively 35%, 35% and 30% of total product. Approximately 1.5%, 1% and 2% of the bulbs produced by these factories are known to be defective. If a randomly selected bulb manufactured by a company was found to be defective, what is the probability that the bulb was manufactured in factory A?

~~31~~ $P(A) = 0.35$ $P(B) = 0.35$
 $P(C) = 0.30$

$$P(D|A) = 0.015$$

$$P(D|B) = 0.01$$

$$P(D|C) = 0.02$$

$$P(A|D) = \frac{P(D|A) * P(A)}{P(D|A) * P(A) + P(D|B) * P(B) + P(D|C) * P(C)}$$

$$= \frac{0.015 * 0.35}{0.015 * 0.35 + 0.01 * 0.35 + 0.02 * 0.30}$$

$$= 0.356$$

Chapter
4

Discrete probability distribution

المتغير العشوائي

★ The Random variable

⇒ A function from the sample space to a set of real numbers (X, Y, Z)

Example when tossing a fair coin

2 - times, the Random variable X is the number of heads obtained, what is the RV?

$$\Omega: \left\{ \begin{array}{l} (H, H), (H, T) \\ (T, H), (T, T) \end{array} \right\}$$

$$X: \{0, 1, 2\}$$

Support

NOTE

Support دعم RV متغير عشوائي

Countable
"integers"
"أرقام منفصلة"

⇒ Discrete Random variable

0, 1, 8, 9

non Countable
"intervals"

"فترات / كسور عشوائية"

⇒ Continuous Random variable

3.8, 6.2

Example

Find probability distribution for

the previous question?

X	0	1	2
P(X=x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

* probability distribution for discrete

Random variable (Discrete probability distribution)

Example Determine whether each Random variable X is discrete or continuous:

① Let X be the number of Fortune 500 Companies that lost money in previous year

② Let X Represent the volume of gasoline in a 21-gallon tank

③ Let X Represent the speed of Rockets

④ Let X Represent the number of Calves born on a farm in one-year

⑤ Let X Represent the number of days of rain for the next 3 days

NOTE

integers

دائماً

Discrete
Random
variable

- قيم

integers

non-integers



Continuous
Random
variable

- قيم

* Probability Mass function (PMF)

$$(1) \quad 0 \leq P(X=x) \leq 1$$

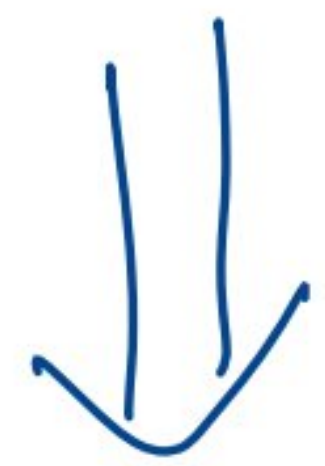
$$(2) \quad \sum P(X=x) = 1$$

Relative Frequency

$$R.F. = \frac{f}{\sum f} = P(X=x)$$

X	0	1	2	3	4	5
f	10	5	1	3	8	3

Frequency
distribution



X	0	1	2	3	4	5
$P(X=x)$	$\frac{10}{30}$	$\frac{5}{30}$	$\frac{1}{30}$	$\frac{3}{30}$	$\frac{8}{30}$	$\frac{3}{30}$

PMF

Example If $P(X=x) = k \cdot x$, $x=1, 2, 3, 4$

and is a PMF, find:

$0 \leq P(X=x) \leq 1$

$\sum (P(X=x)) = 1$

x	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$4k$

① The value of k

$$k + 2k + 3k + 4k = 1$$

$$10k = 1$$

$$k = 0.1$$

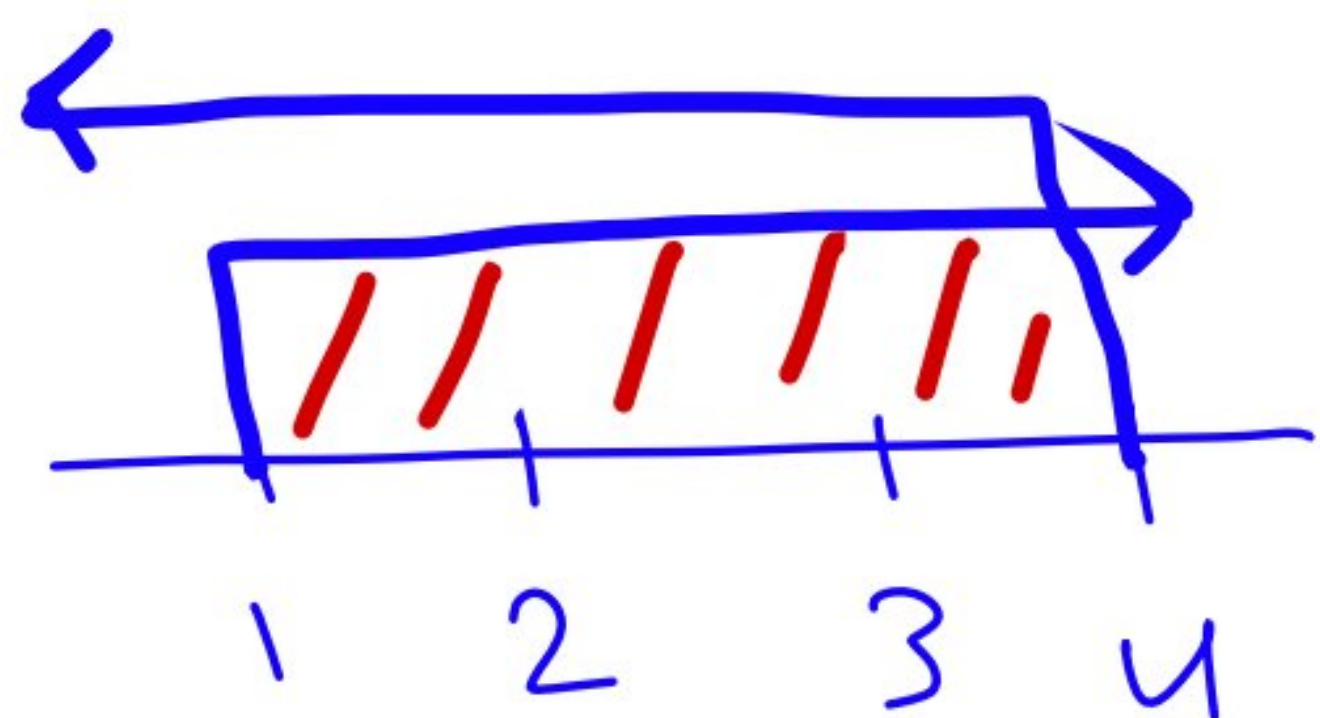
② $P(X=3) = 3k = 3 \times 0.1 = 0.3$

③ $P(1 < X \leq 4)$



$$= P(X=2) + P(X=3) + P(X=4) = 0.9$$

④ $P(X > 1 \mid X < 4) = \frac{P(X > 1 \cap X < 4)}{P(X < 4)}$



$$= \frac{P(X=2) + P(X=3)}{P(X=1) + P(X=2) + P(X=3)}$$

$$= \frac{0.5}{0.6} = \frac{5}{6}$$

$$\textcircled{5} P(X = 3.4) = 0$$

Example when throwing a fair dice 2

times, define the Random variable S to be the sum of 2 numbers obtained. Find the

Probability distribution of S ?

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

S	2	3	4	5	6	7	8	9	10	11	12
$P(S=s)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



Symmetric

μ . population mean

\bar{x} . sample mean

$$\boxed{\mu = 7}$$

mean

* The Expected value ($E(x)$)

$$E(x) = \mu = \text{mean} = \sum x \cdot P(x=x)$$

Example find the expected value for.

①	x	1	2	3	4
	$P(x=x)$	0.4	0.3	0.2	0.1

$$E(x) = 1 \times 0.4 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.1$$
$$= 2$$

②	x	1	2	3
	$P(x=x)$	A	B	A

$$\mu = E(x) = 2 \Rightarrow \text{Symmetric}$$

Example If the $E(x) = 2$, find a and b ?

x	1	2	3	4
$P(x=x)$	a	0.3	b	0.1

sol $2 = 1 \times a + 2 \times 0.3 + 3 \times b + 4 \times 0.1$

$$2 = a + 0.6 + 3b + 0.4$$

←
 $2 = 1 + a + 3b$

$$\boxed{a + 3b = 1} \dots \textcircled{1}$$

$$a + 0.3 + b + 0.1 = 1$$

$$a + b + 0.4 = 1$$

$$\boxed{a + b = 0.6} \dots \textcircled{2}$$

$$\begin{array}{r} - a + b = 0.6 \\ - a + 3b = 1 \end{array}$$

$$a + 0.2 = 0.6$$

$$\boxed{a = 0.4}$$

$$-2b = -0.4$$

$$b = 0.2$$

* Properties of $E(x)$:

$$\textcircled{1} E(a) = a$$

الثابت نفسه

$$E(3) = 3$$

$$E(2.5) = 2.5$$

$$\textcircled{2} E(ax) = a * E(x) \Rightarrow E(2x) = 2 * E(x)$$

$$\textcircled{3} E(x \pm y) = E(x) \pm E(y)$$

Example

x	1	2	3	4
$P(x=x)$	0.4	0.3	0.2	0.1

find:

$$\textcircled{1} E(x) = 1 * 0.4 + 2 * 0.3 + 3 * 0.2 + 4 * 0.1$$
$$= 2$$

$$\textcircled{2} E(x^2) = 1^2 * 0.4 + 2^2 * 0.3 + 3^2 * 0.2$$

$$+ 4^2 * 0.1$$

$$= 5$$

$$\textcircled{3} E\left(\frac{1}{x}\right) = \frac{1}{1} * 0.4 + \frac{1}{2} * 0.3$$

$$+ \frac{1}{3} * 0.2 + \frac{1}{4} * 0.1$$

$$= \text{---}$$

$$\textcircled{4} E(e^x) = e^1 * 0.4 + e^2 * 0.3 + e^3 * 0.2$$

$$+ e^4 * 0.1 = \text{---}$$

العدد الطبيعي
 ≈ 2.7

Example If the $\mu = E(x) = 10$, find:

$$\textcircled{1} E(\mu) = E(10) = 10$$

$$\textcircled{2} E(E(x)) = E(10) = 10$$

$$\begin{aligned} \textcircled{3} \quad E(2x-3) &= E(2x) - E(3) \\ &= 2 * E(x) - 3 \\ &= 2 * 10 - 3 = 17 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad E\left(1 - \frac{x}{2}\right) &= E(1) - E\left(\frac{x}{2}\right) \\ &= 1 - E\left(\frac{1}{2} * x\right) \\ &= 1 - \frac{1}{2} * E(x) = -4 \end{aligned}$$

^{مثال}
Example If the probability mass function for the number of episodes of otitis media in the first 2 years of life are shown, find the Expected number of visits?

r	0	1	2	3	4	5	6
$P(R=r)$	0.129	0.264	0.271	0.185	0.095	0.039	0.017

$$E(X) = 0 * 0.129 + 1 * 0.264 + 2 * 0.271 + 3 * 0.185 + 4 * 0.095 + 5 * 0.039 + 6 * 0.017 \approx 2.0 \dots$$

مثال

Example Determine which one of the following

tables represent PMF:

A)

v	0	1	2	3
$P(X=v)$	0.18	0.34	0.36	0.13

X

$$\sum P(X=x) = 1.01$$

B)

v	0	1	2	3
$P(X=v)$	0.10	0.42	0.03	0.44

X

C)

v	0	1	2	3
$P(X=v)$	0.12	0.13	0.61	0.14



D)

v	0	1	2	3
$P(X=v)$	0.26	0.44	-0.15	0.45

X

A)

v	0	1	2	3	4
$P(X=v)$	0.15	0.25	0.10	0.25	0.30

(X)

B)

v	0	1	2	3
$P(X=v)$	0.15	0.20	0.30	0.10

(X)

C)

v	0	1	2	3	4
$P(X=v)$	0.15	-0.20	0.30	0.20	0.15

(X)

D)

v	-1	0	1	2	3	4
$P(X=v)$	0.15	0.30	0.20	0.15	0.10	0.10

(✓)

★ The variance (σ^2)

$$\sigma^2 = \text{Var}(X) = E(X - \mu)^2$$

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$\sigma^2 = E(X^2) - \mu^2$$

Example

X	1	2	3	4
P(X=x)	0.4	0.3	0.2	0.1

Find the variance and STD?

$$\sigma^2 = E(X^2) - \mu^2$$

$$\Rightarrow E(X^2) = 1^2 * 0.4 + 2^2 * 0.3 + 3^2 * 0.2 + 4^2 * 0.1$$
$$= 5$$

$$\Rightarrow E(X) = \mu = 1 * 0.4 + 2 * 0.3 + 3 * 0.2 + 4 * 0.1$$
$$= 2$$

$$\sigma^2 = 5 - (2)^2 = 1$$

$$\sigma = \sqrt{1} = 1$$

★ Properties of variance:

$$\textcircled{1} \text{Var}(a) = 0$$

$$\text{Var}(9) = 0, \text{Var}(3) = 0$$

$$\textcircled{2} \text{Var}(ax) = a^2 * \text{Var}(x)$$

$$\textcircled{3} \text{Var}(ax \pm b) = \text{Var}(ax) \pm \cancel{\text{Var}(b)} \\ = a^2 * \text{Var}(x)$$

$$\textcircled{4} \text{Var}(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \text{Var}(x) + (E(x))^2$$

$$E(x^2) = \sigma^2 + \mu^2$$

∴
Example If $\mu = 10$, $\sigma^2 = 3$ find:

$$i) E(\text{Var}(x)) = E(3) = 3$$

$$\text{ii) } \text{Var}(E(x)) = \text{Var}(10) = 0$$

$$\text{iii) } \text{Var}(\text{Var}(x)) = \text{Var}(3) = 0$$

$$\text{iv) } E(x - 10)^2 = E(x^2 - 20x + 100)$$

NOTE
 $(a \pm b)^2 = a^2 \pm 2ab + b^2$

$$E(x^2) - 20 * E(x) + E(100)$$

$$\sigma^2 + \mu^2 - 20 * 10 + 100$$

$$3 + 100 - 200 + 100 = \textcircled{3}$$

دلیل

$$\text{Var}(x) = E(x - \mu)^2$$

$$= E(x - \mu)^2 = \text{Var}(x) = \textcircled{3}$$

$$\text{v) } E(x^2) = \sigma^2 + \mu^2 = 3 + 10^2 = 103$$

$$\text{vi) } E(x - 2)^2 = E(x^2 - 4x + 4)$$

$$= E(x^2) - u * E(x) + E(u)$$

$$= 103 - 40 + u$$

$$= \textcircled{67}$$

r	0	1	2	3	4	5	6	PMF
$P(R=r)$	0.129	0.264	0.271	0.185	0.095	0.039	0.017	

NOTE: تقریباً 95% من البيانات

في PMF تقع بين 2 و 6

$E(x)$

$$E(x) = 2.038$$

$$\sigma = 1.402$$

find a & b such that approximately 95% of data lie within it?

$$\begin{array}{c} 95\% \\ \hline \left(\begin{array}{c} 25 \\ \hline a \end{array} \quad \begin{array}{c} 25 \\ \hline 2.038 \end{array} \quad \begin{array}{c} 25 \\ \hline b \end{array} \right) \end{array}$$

$$a = 2.038 - 2 \times 1.402 = -$$

$$b = 2.038 + 2 \times 1.402 = -$$

★ Binomial distribution

توزيع ثنائي

★ If we have n -independent trials $n \geq 2$ and the outcomes in EACH trial are Success or fail only. Let X be the number of Success, then we say that X follows binomial distribution and is denoted by $X \sim \text{Bin}(n, p)$, where:

n : number of trials

p : probability of success

توزيع ثنائي Binomial

(1) $n \geq 2$

② outcomes $\begin{cases} \rightarrow \text{Success} \\ \rightarrow \text{fail} \end{cases}$

نتيجة فقط

③ independent

$X \sim \text{Bin}(n, P)$
follows
يتبع

n : number of trials

q : Probability of fail in each trial

P : probability of success in each trial

~~قوانين~~

① $P + q = 1$

② $P(X = k) = \binom{n}{k} * P^k * q^{n-k}$

$\binom{n}{k}$: Combination

③ $\mu = E(X) = n \cdot P$

④ $\text{var}(X) = \sigma^2 = n \cdot P \cdot q$

$$\textcircled{5} \text{ STD}(X) = \sigma = \sqrt{n \cdot p \cdot q}$$

Example when tossing a fair coin 10 times, find:

① independent
 ② $n \geq 2$
 ③ outcomes $\begin{matrix} \nearrow H \\ \searrow T \end{matrix}$

$X \sim \text{Bin}(10, 0.5)$

A) the probability of getting:

i) exactly 8 heads

$$P(X=8) = \binom{10}{8} * 0.5^8 * 0.5^{(10-8)}$$

$$= \underline{\underline{0.04}}$$

ii) at least 9 H = $P(X \geq 9)$

$$= P(X=9) + P(X=10)$$

$$= \binom{10}{9} * 0.5^9 * 0.5^{(10-9)} + \dots$$

at least :	على الأقل	$P(X \geq k)$
at most :	على الأكثر	$P(X \leq k)$

iii) at least 2 H

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - (P(X=0) + P(X=1))$$

:

iv) at most 1 H

$$P(X \leq 1) = (P(X=0) + P(X=1))$$

v) at most 8 H

$$P(X \leq 8) = 1 - P(X > 8)$$

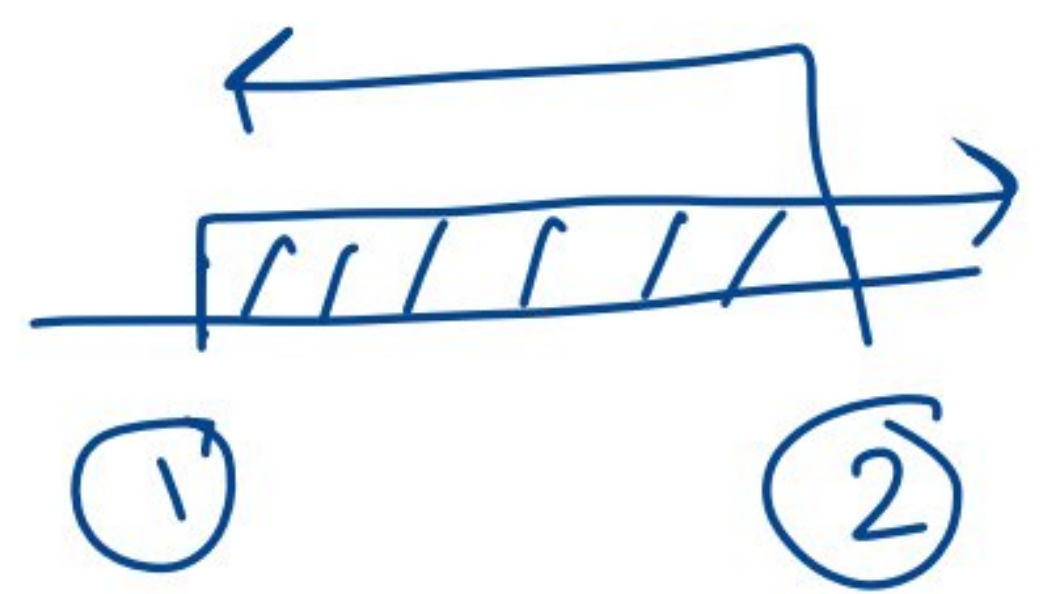
$$= 1 - P(X \geq 9)$$

$$= 1 - (P(X=9) + P(X=10))$$

vi) at most 2 H given that at least 1 H.

$$\Rightarrow P\left(\begin{array}{c} \text{at most} \\ 2H \end{array} \middle| \begin{array}{c} \text{at least} \\ 1H \end{array}\right) = \frac{P(\text{at most } 2H \cap \text{at least } 1H)}{P(\text{at least } 1H)}$$

$$= \frac{P(X \leq 2 \cap X \geq 1)}{P(X \geq 1)}$$



$$= \frac{P(X=1) + P(X=2)}{(1 - P(X < 1))}$$

$$= \frac{P(X=1) + P(X=2)}{(1 - P(X=0))} = \dots$$

B) The Expected number of heads and the variance and the standard deviation

$$\Rightarrow E(X) = \mu = n \cdot p$$

$$= 10 * 0.5 = 5$$

$$\Rightarrow \sigma^2 = n \cdot p \cdot q = 10 * \frac{1}{2} * \frac{1}{2} = 2.5$$

$$\Rightarrow \sigma = \sqrt{n \cdot p \cdot q} = \sqrt{2.5} = \text{---}$$

^{9,1}
Example If $X \sim \text{Bin}(3, p)$ and $P(X \geq 1) = \frac{19}{27}$
 find $\text{var}(X)$?

~~8,1~~

$$\text{var}(X) = n \cdot p \cdot q$$

$$= 3 * \frac{1}{3} * \frac{2}{3} = \boxed{\frac{2}{3}}$$

$$P(X \geq 1) = \frac{19}{27}$$

$$1 - P(X < 1) = \frac{19}{27}$$

$$1 - \frac{19}{27} = P(X = 0)$$

$$\frac{8}{27} = \binom{3}{0} * p^0 * q^{(3-0)}$$

$$\frac{8}{27} = q^3 \Rightarrow \boxed{q = \frac{2}{3}} \quad \boxed{p = \frac{1}{3}}$$

Example If $X \sim \text{Bin}(n, p)$ and $\mu = 2, \sigma^2 = 1.6$
find n, p ?

$$\begin{aligned} 2 &= n \cdot p & \longrightarrow & 2 = n * \frac{2}{10} \\ 1.6 &= \frac{n \cdot p \cdot q}{1} & & \boxed{n = 10} \\ & \downarrow & & \\ 1.6 &= 2 * q \end{aligned}$$

$$\boxed{q = 0.8} \quad \boxed{p = 0.2}$$

* Using the Binomial tables

Example If $X \sim \text{Bin}(10, 0.4)$, Find.

i) $P(X \leq 6) = 0.945$

ii) $P(X > 6) = 1 - P(X \leq 6)$
 $= 1 - 0.945 = \text{---}$

$$\text{iii) } P(X < 7) = P(X \leq 6) = 0.945$$

$$\begin{aligned} \text{iv) } P(X \geq 7) &= 1 - P(X < 7) \\ &= 1 - P(X \leq 6) \\ &= 1 - 0.945 = \text{---} \end{aligned}$$

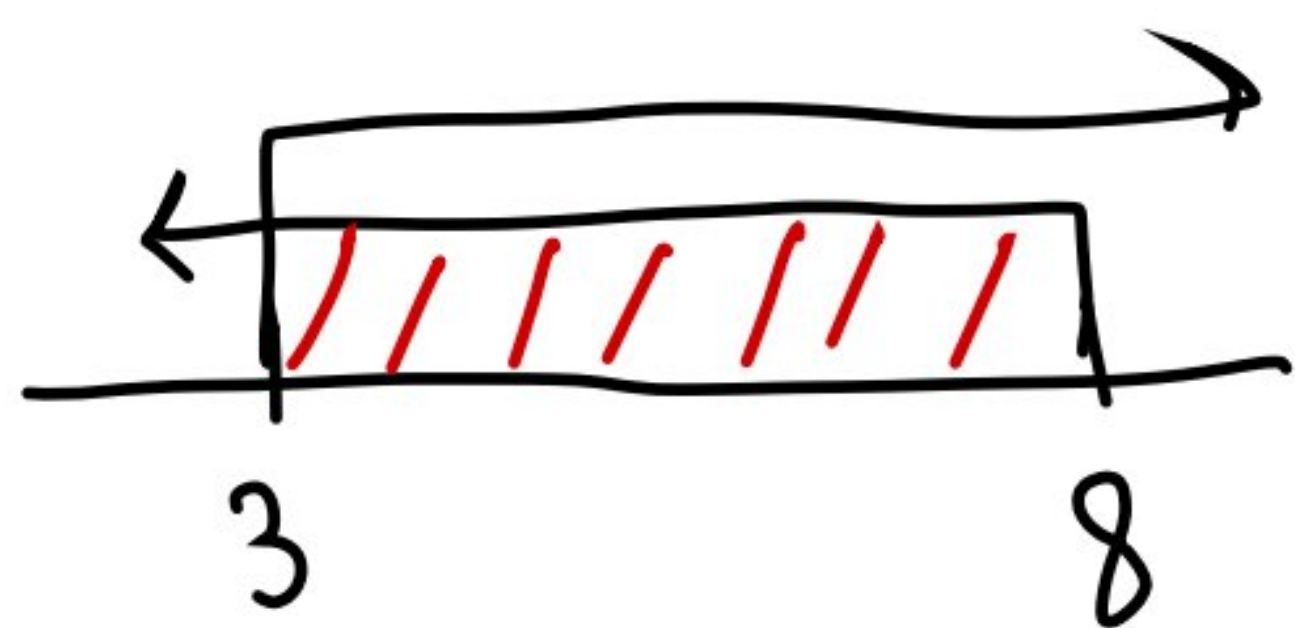
$$\text{v) } P(3 \leq X \leq 8) =$$

$$\text{vi) } P(3 < X \leq 8) = P(4 \leq X \leq 8)$$

$$\text{vii) } P(3 < X < 8) = P(4 \leq X \leq 7)$$

$$\text{viii) } P(3 \leq X < 8) = P(3 \leq X \leq 7)$$

$$\text{ix) } P(X \geq 3 | X \leq 8) = \frac{P(X \geq 3 \cap X \leq 8)}{P(X \leq 8)}$$



$$= \frac{P(3 \leq X \leq 8)}{P(X \leq 8)}$$

Example A family has 5 children, what is the probability that 3 children are females.

$$X \sim \text{Bin}(5, 0.5)$$

$$P(X=3) = \binom{5}{3} * 0.5^3 * 0.5^{(5-3)} = 0.3125$$

Example In a multiple choice exam of 10-quest. each question had 5 answers only one of them is correct. Ahmad is answering the exam by guessing ^{التخمين} what is the probability that ahmad will answer:

1) 5 questions correctly $X \sim \text{Bin}(10, 0.2)$

$$P(X=5) = \binom{10}{5} * 0.2^5 * 0.8^{(10-5)} = 0.027$$

2) at most 5 questions

$$P(X \leq 5) = 0.994$$

Ex Example what is the probability of obtaining 2 boys out of 5 children if the probability of a boy is ^P0.51 at each birth and the genders are considered independent Random variables?

$$X \sim \text{Bin}(5, 0.51)$$

$$P(X=2) = \binom{5}{2} * 0.51^2 * 0.49^{(5-2)}$$

$$= 0.306$$

Ex Example Evaluate the probability of 2 lymphocytes out of 10 white blood cells if the probability of any one cell being a lymphocyte is ^P0.2.

$$X \sim \text{Bin}(10, 0.2)$$

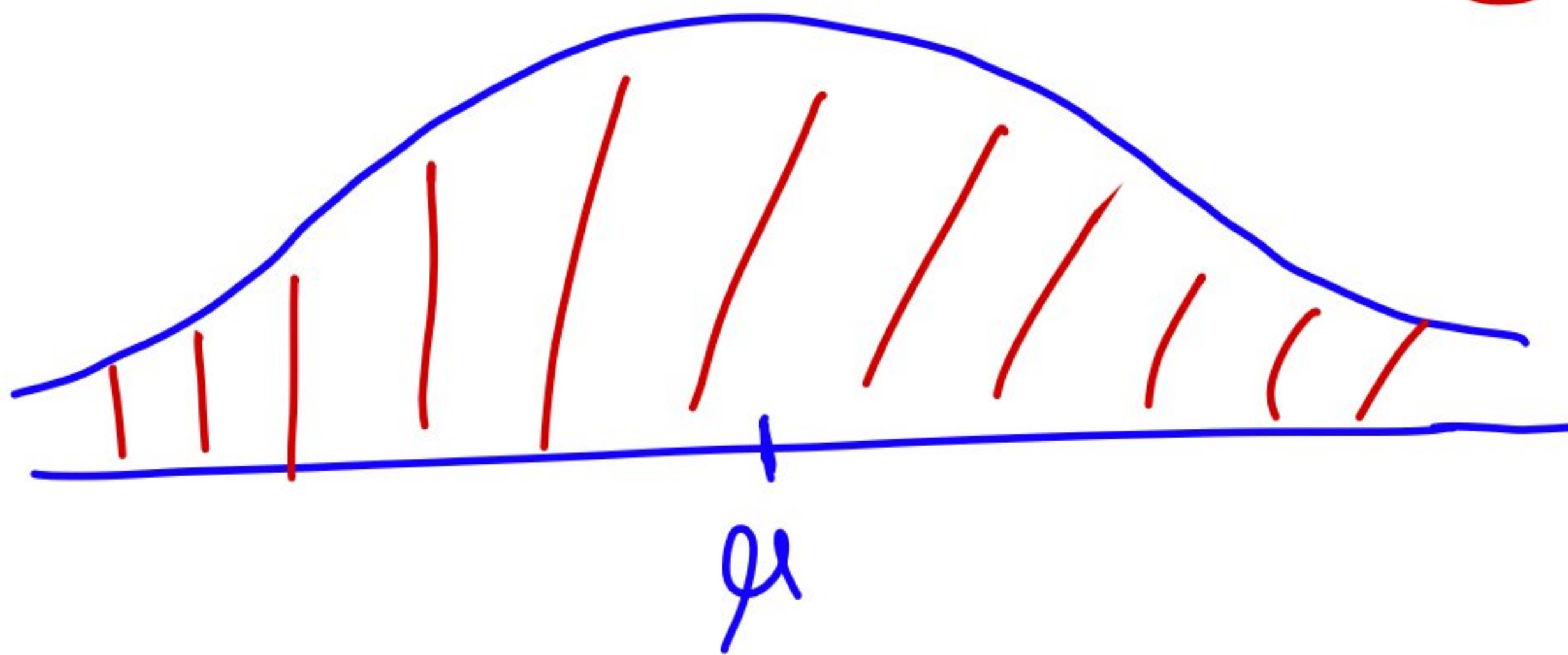
$$P(X=2) = \binom{10}{2} * 0.2^2 * 0.8^{10-2} = 0.3020$$

Chapter
5

The normal distribution

التوزيع الطبيعي

→ Continuous probability distribution



① Symmetric

② $\mu = \text{mode} = Q_2$

③ Total area = 1

④ ^{مساحة} probability = area

⑤ $P(X=k) = 0$

⑥ $P(X \leq k) = P(X < k)$

ملاحظة لا علاقة ليس لها معنى في

normal distribution

$$X \sim N(\mu, \sigma^2)$$

- follows

- is distributed as

Examples of normal distribution:

- weights

- speed

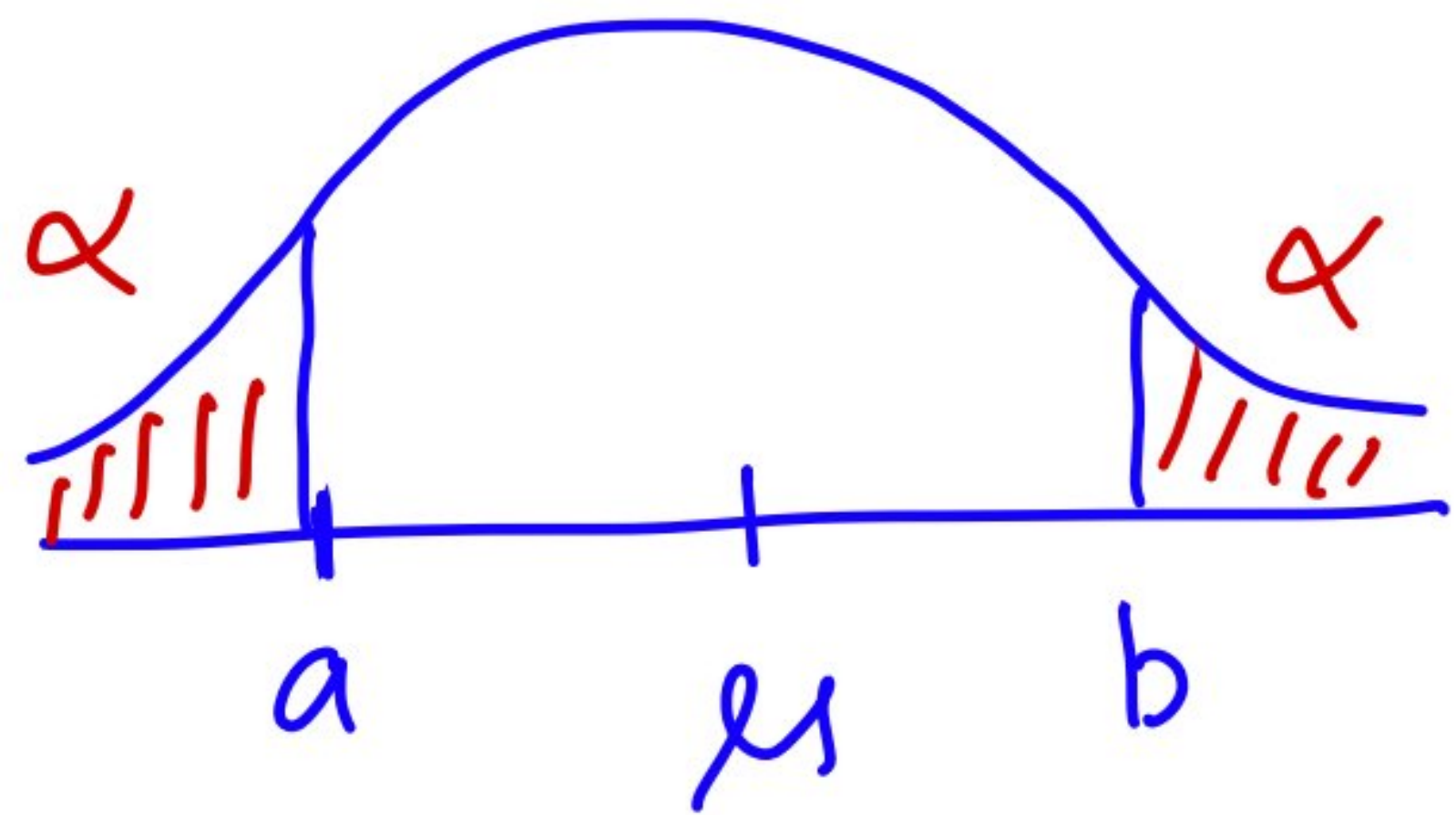
- Blood pressure

- height

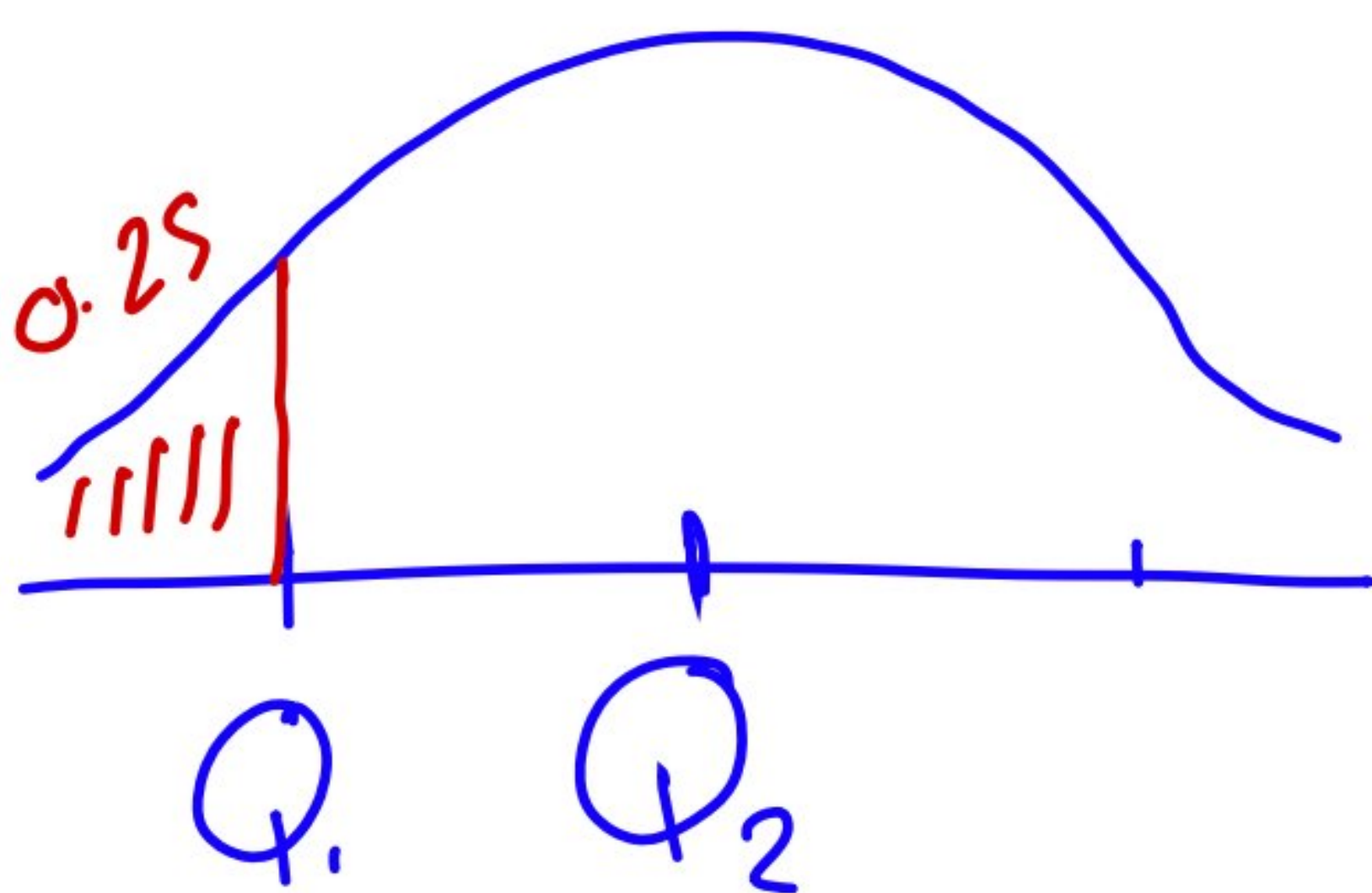
NOTES

①

$$\mu = \frac{a+b}{2}$$



②

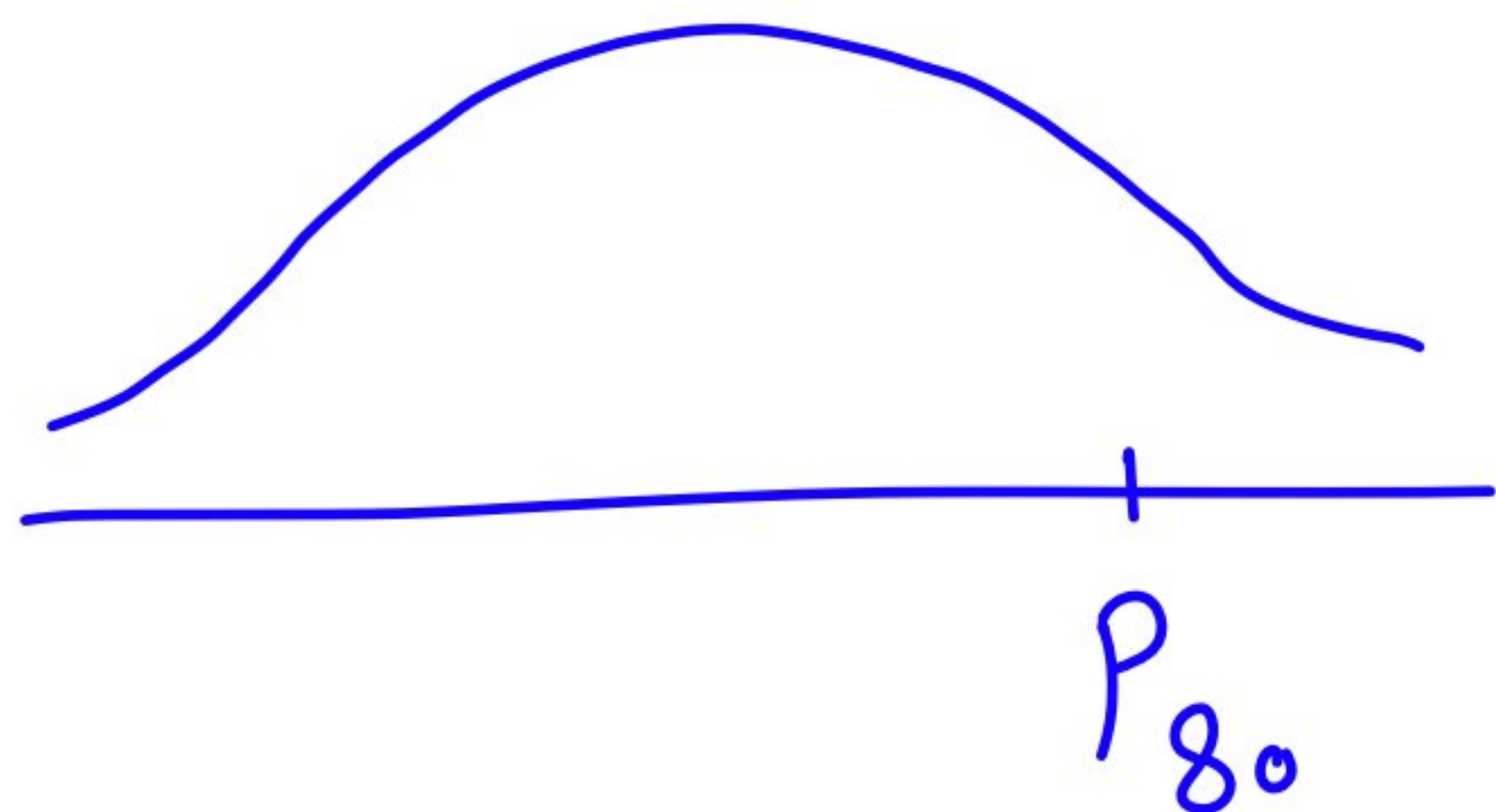


$$- P(X \leq Q_1) = 0.25$$

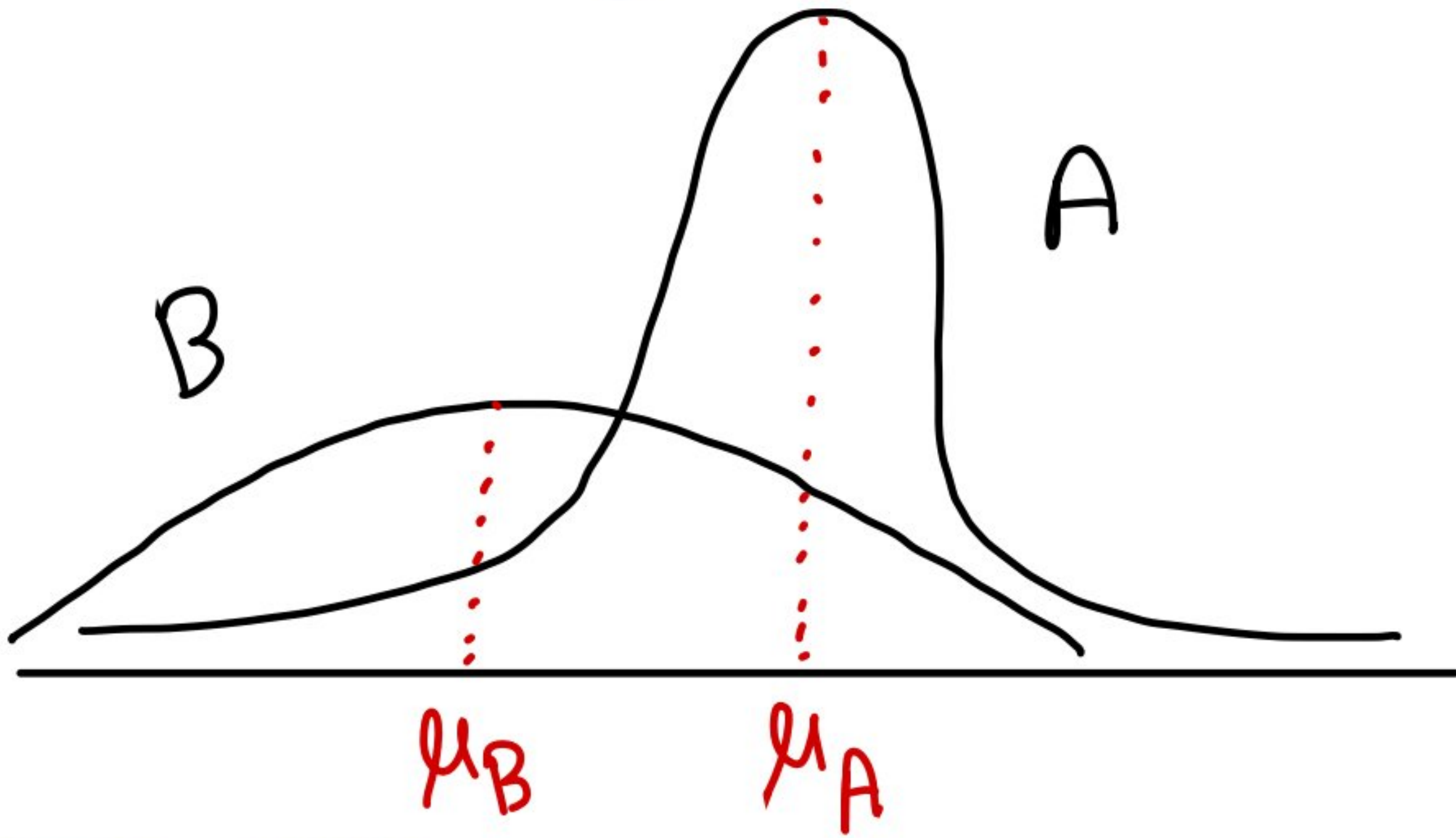
$$- P(X \leq Q_3) = 0.75$$

③

$$P(X \leq P_{80}) = 0.80$$

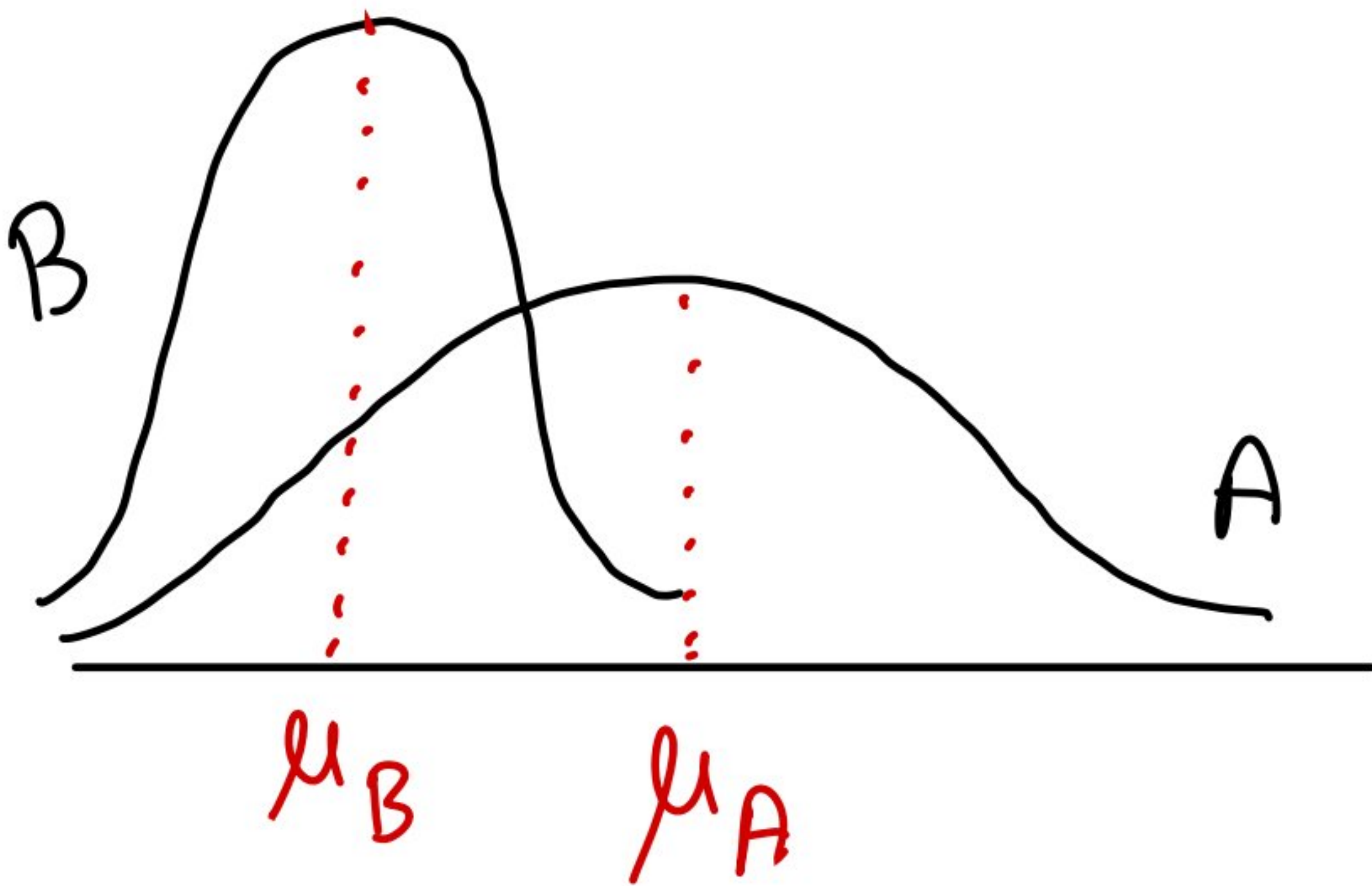


Example A & B are normally distributed curves. which one has larger mean? which one has larger standard deviation?



$$\mu_A > \mu_B$$

$$SD_B > SD_A$$



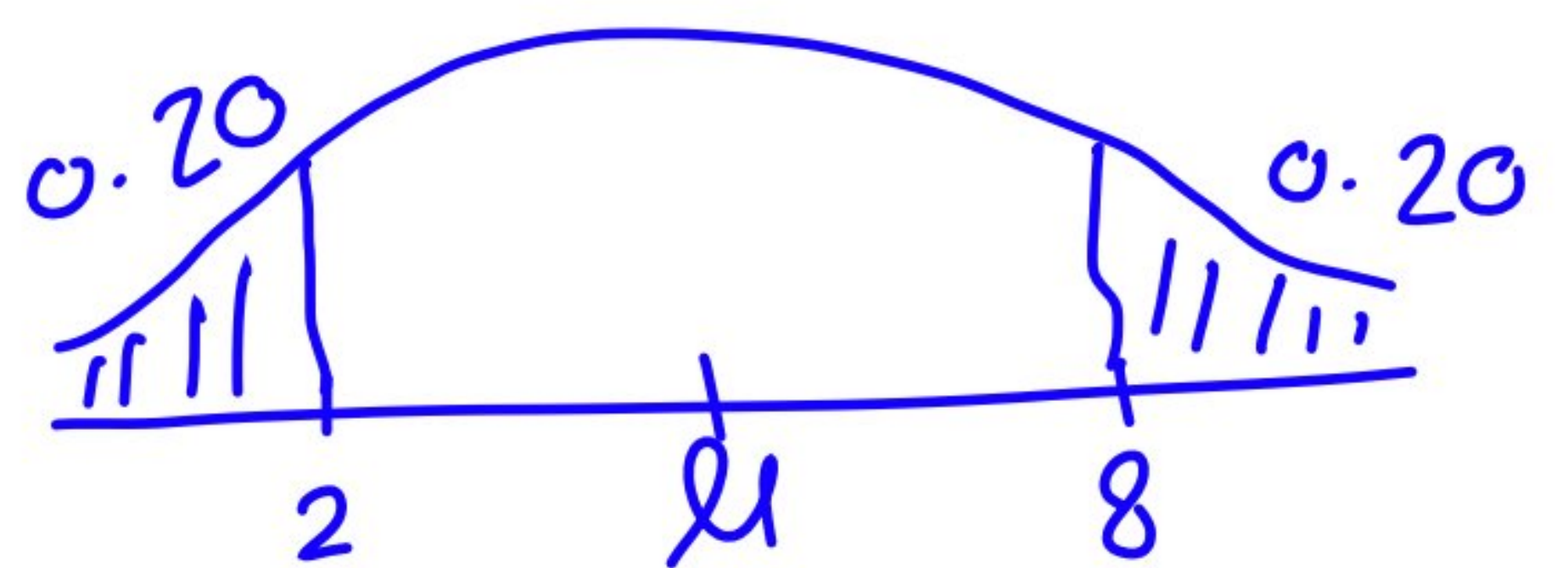
$$\mu_A > \mu_B$$

$$SD_A > SD_B$$

Example If $X \sim n(\mu, \sigma)$, and

$P(X < 2) = 0.20$, $P(X > 8) = 0.20$, Find:

① $\mu = \frac{2 + 8}{2} = 5$



$$\textcircled{2} P(2 \leq X < 8)$$

$$0.20 + X + 0.20 = 1$$

$$X + 0.40 = 1$$

$$X = 0.60$$

$\textcircled{3}$ 20th percentile (P_{20})

$$P(X \leq P_{20}) = 0.20$$

$$P_{20} = 2$$

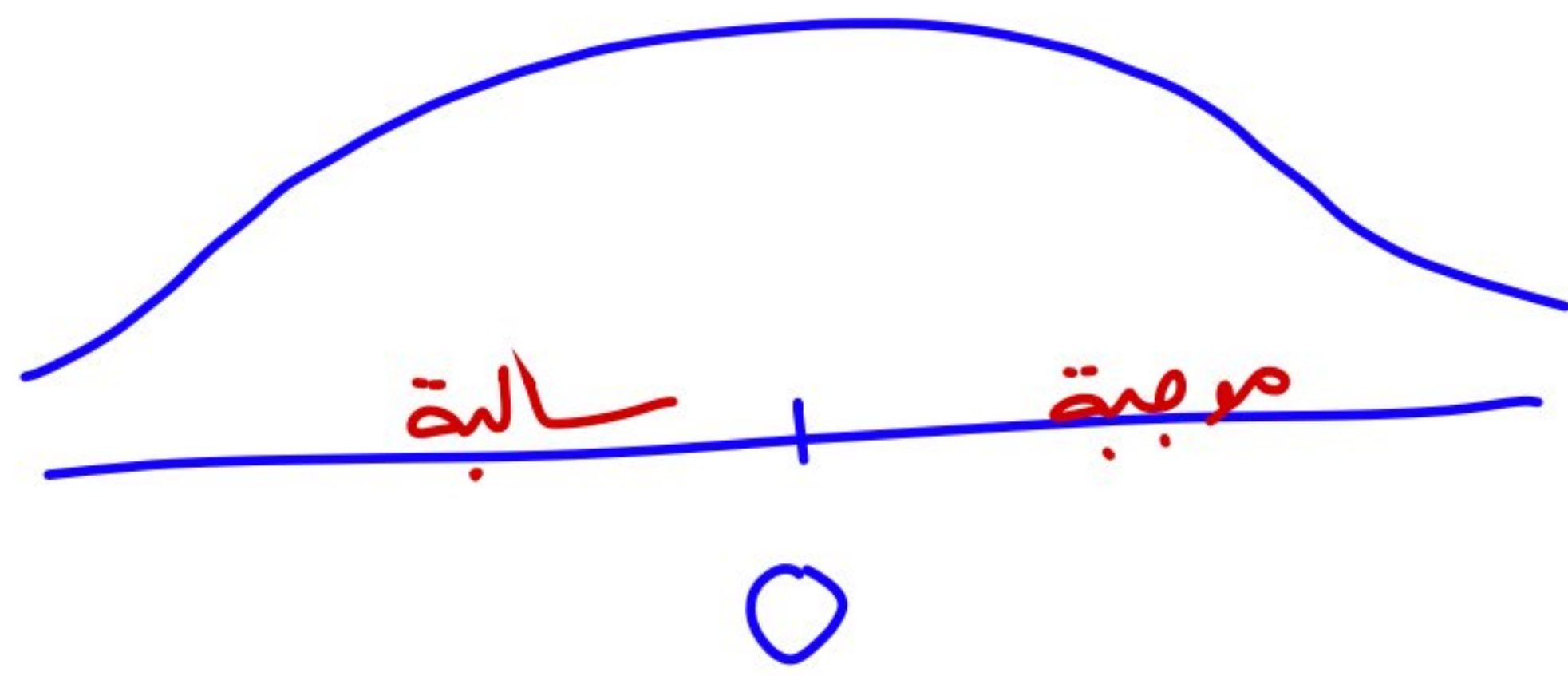
$\textcircled{4}$ 80th percentile (P_{80})

$$P(X \leq P_{80}) = 0.80$$

$$P_{80} = 8$$

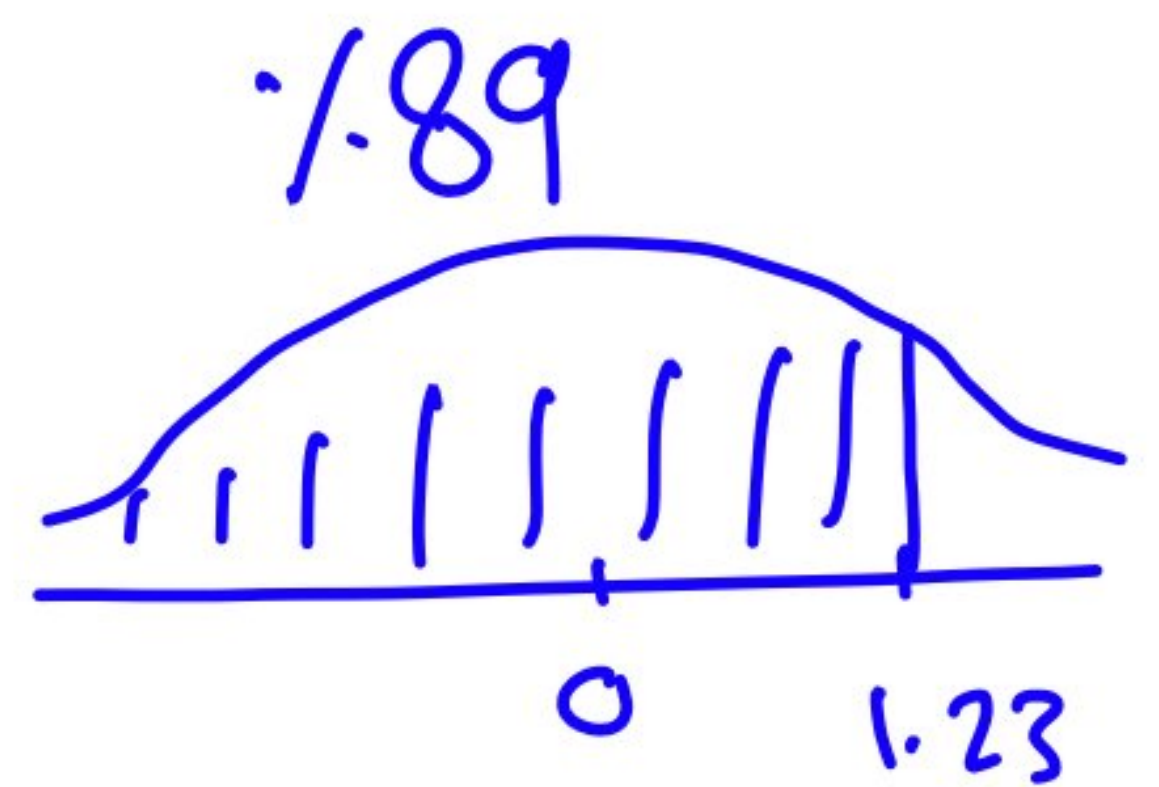
* The standard normal distribution

$$Z \sim N(0, 1)$$



Example If $Z \sim N(0,1)$, then find:

i) $P(Z \leq 1.23) = 0.8907$



ii) $P(Z \geq 1.23) = 0.1093$

iii) $P(1.23 \leq Z \leq 2.12)$

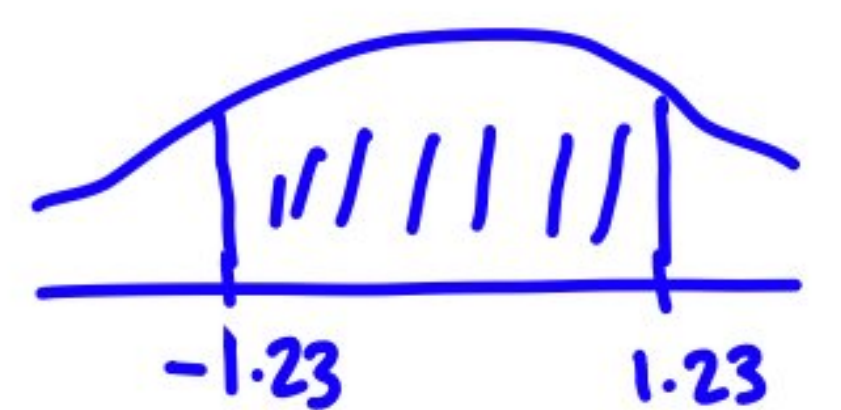
$$= P(Z \leq 2.12) - P(Z \leq 1.23)$$

$$= 0.9830 - 0.8907$$

iv) $P(-1.23 < Z < 1.23)$

$$= P(Z < 1.23) - P(Z < -1.23)$$

$$= 0.8907 - 0.1093 = \text{---}$$



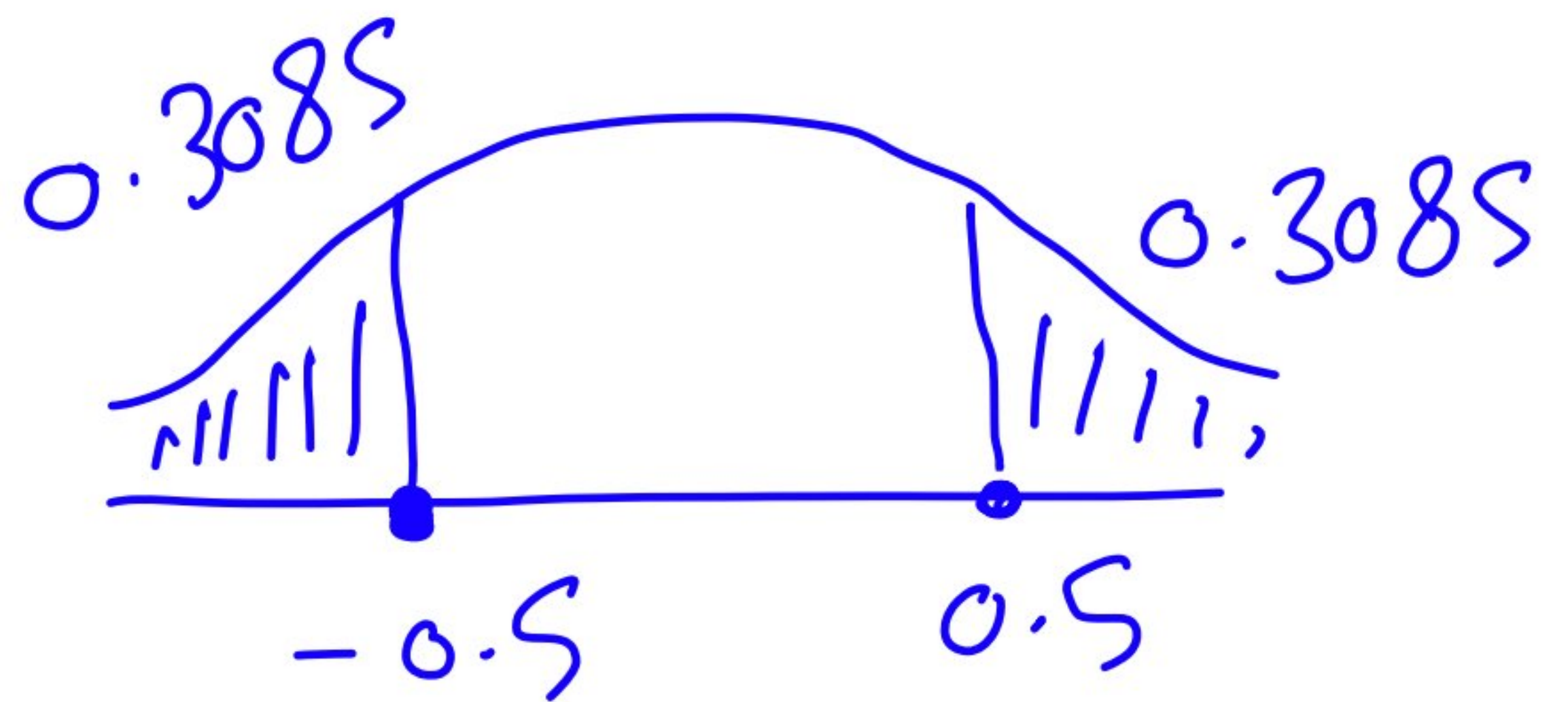
v) The z-score that correspond to a cumulative area of 0.6915

$$z = 0.5$$



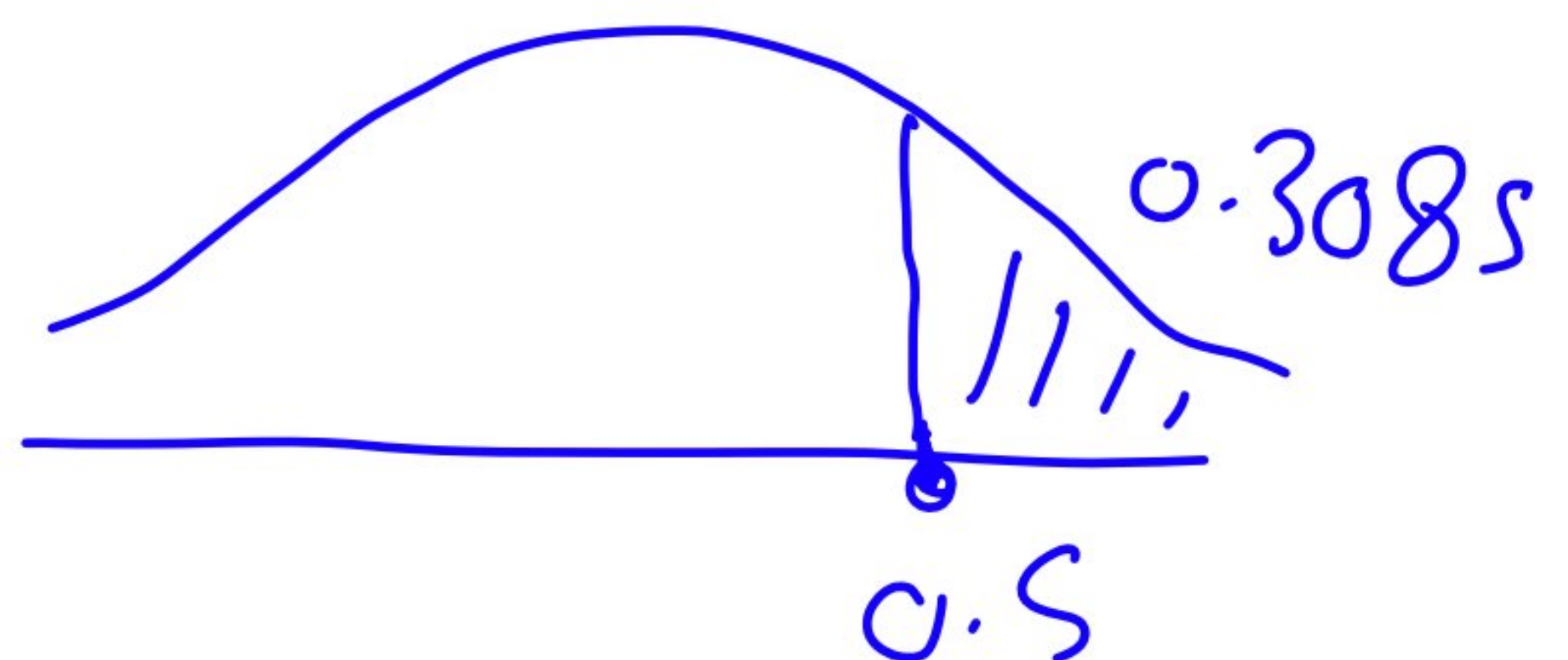
vi) If $P(Z \leq a) = 0.3085$ find a ?

$$a = -0.5$$



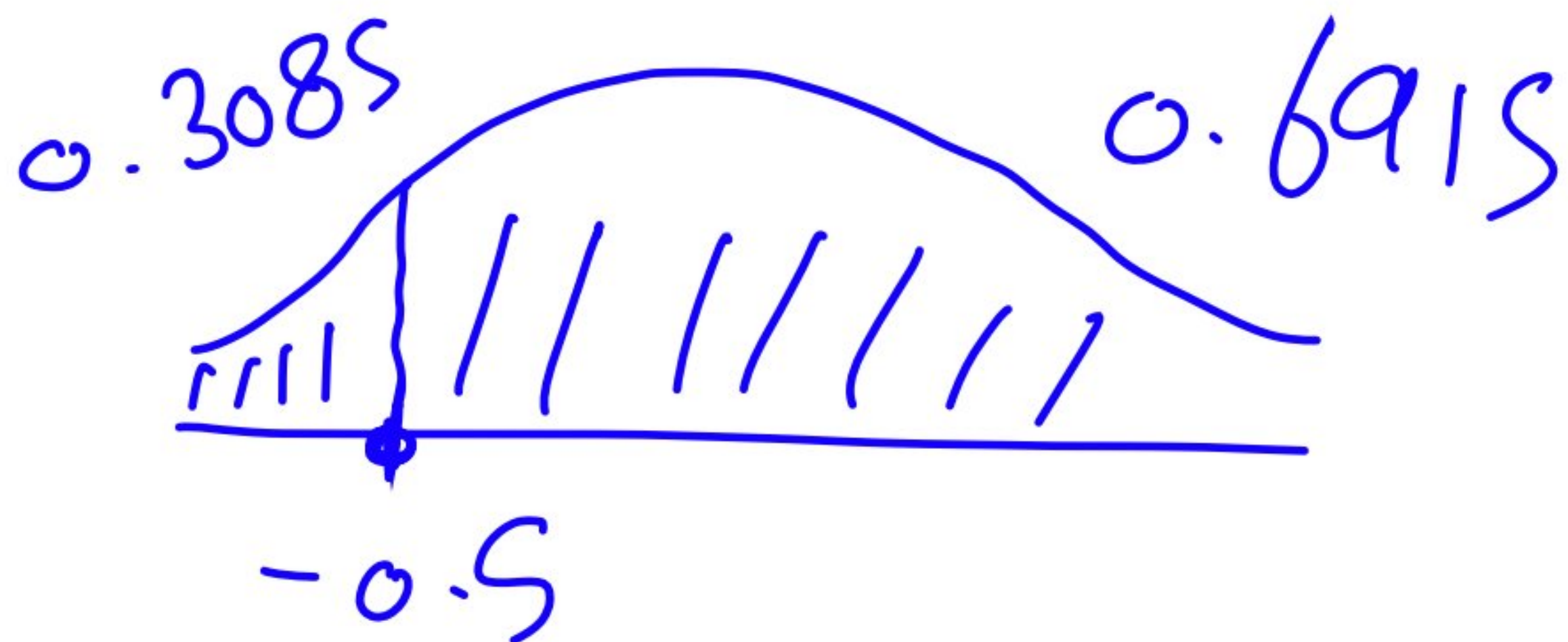
vii) a if $P(Z > a) = 0.3085$?

$$a = 0.5$$



viii) a if $P(Z \geq a) = 0.6915$

$$a = -0.5$$



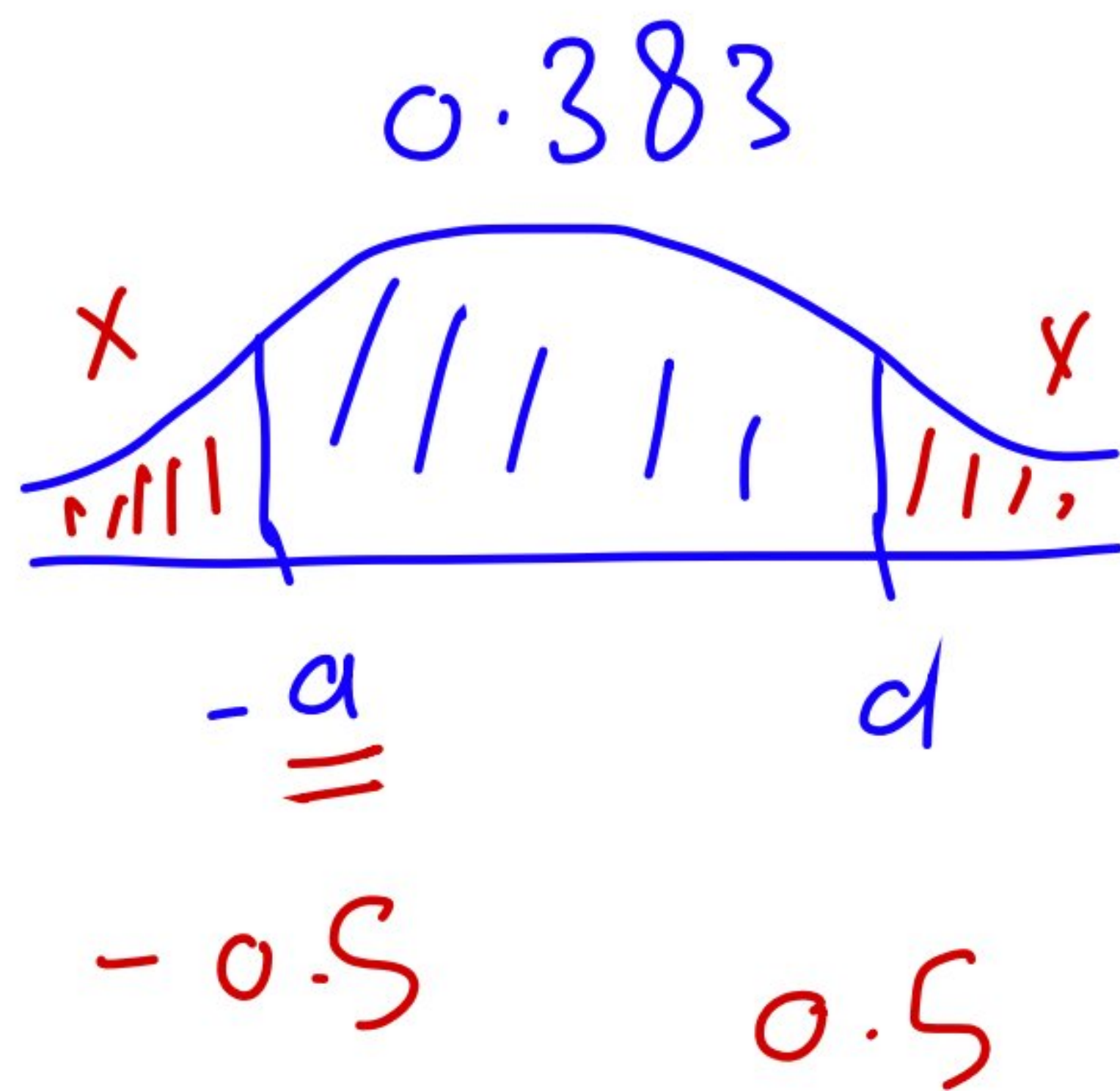
ix) a if $P(-a < Z < a) = 0.383$

$$P(Z < a) - P(Z < -a) = 0.383$$

$$X + X + 0.383 = 1$$

$$X = 0.3085$$

$$a = \pm 0.5$$



* Standardization

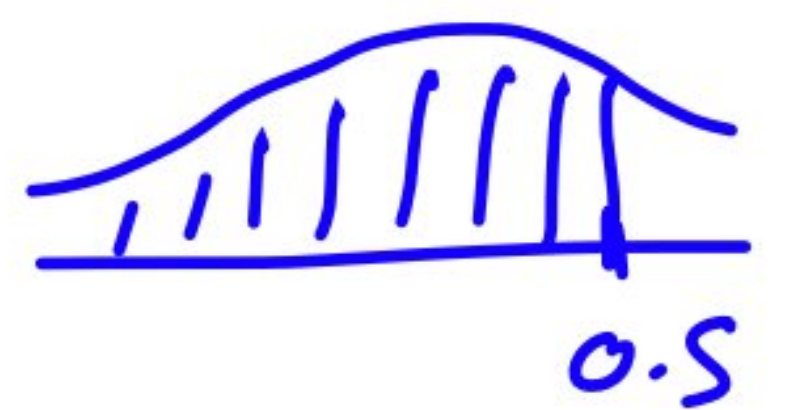
$$\textcircled{n} \longrightarrow \textcircled{Z}$$

$$X \sim n(\mu, \sigma^2)$$

$$\Rightarrow Z = \frac{X - \mu}{\sigma} \sim n(0, 1)$$

Example If $X \sim n(5, 4)$, find:

1) $P(X \leq 6)$



$$= P\left(Z \leq \frac{6 - 5}{2}\right) = P(Z \leq 0.5)$$

$$= 0.6915$$

2) $P(X > 4) \Rightarrow P\left(Z > \frac{4 - 5}{2}\right)$

$$\Rightarrow P(Z > -0.5)$$

$$= 0.6915$$

$$3) P(4 \leq X \leq 6)$$

$$= P(X \leq 6) - P(X \leq 4)$$

$$= P\left(Z \leq \frac{6-5}{2}\right) - P\left(Z \leq \frac{4-5}{2}\right)$$

$$= P(Z \leq 0.5) - P(Z \leq -0.5)$$

$$= 0.6915 - 0.3085$$

$$4) a \text{ if } P(X > a) = 0.6915$$

$$P\left(Z > \frac{a-5}{2}\right) = 0.6915$$

$$P(Z > -0.5) = 0.6915$$

$$-0.5 = \frac{a-5}{2}$$

$$\boxed{a = 4}$$

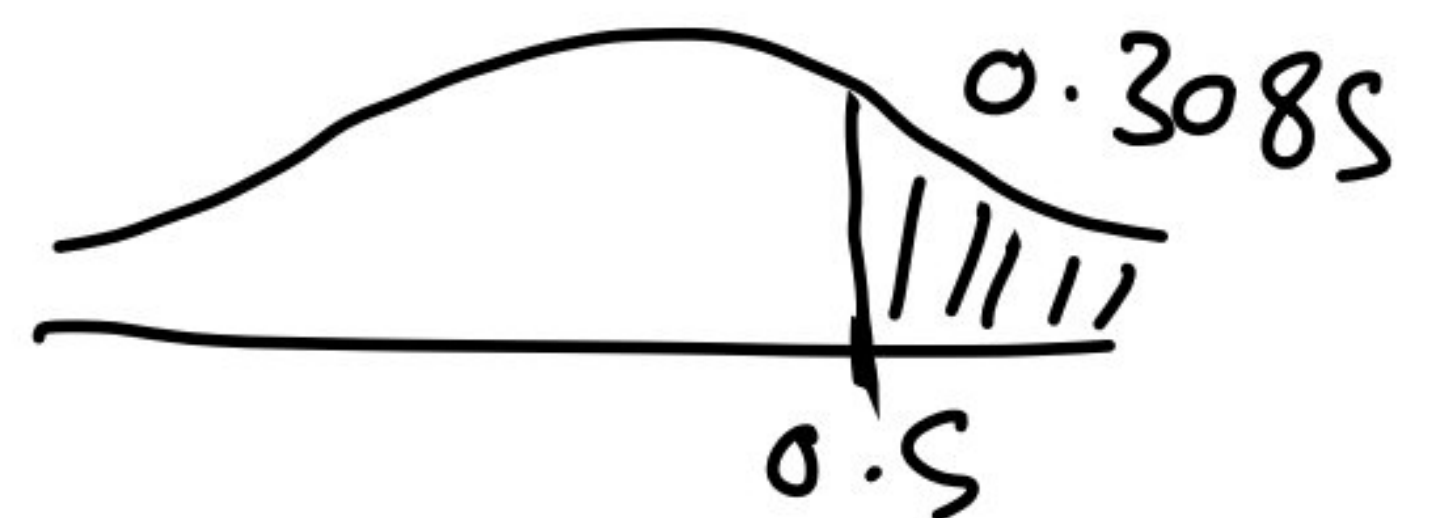
Example If $X \sim N(\mu, \sigma)$, $P(X \geq 6) = 0.3085$

find μ ?

$$\Rightarrow P(X \geq 6) = 0.3085$$

$$= P\left(Z \geq \frac{6 - \mu}{\sigma}\right) = 0.3085$$

$$\frac{1}{\cancel{\sigma}} = \frac{6 - \mu}{\cancel{\sigma}}$$



$$1 = 6 - \mu$$

$$\boxed{\mu = 5}$$

Example If $X \sim N(5, \sigma^2)$,

$P(X > 4) = 0.6915$ find σ^2 ?

$$\Rightarrow P(X > 4) = 0.6915$$

$$= P\left(Z > \frac{4 - 5}{\sigma}\right) = 0.6915$$

$$-\frac{1}{\sigma} = \frac{-1}{\sigma}$$



$$\sigma = 2$$

$$\sigma^2 = 4$$

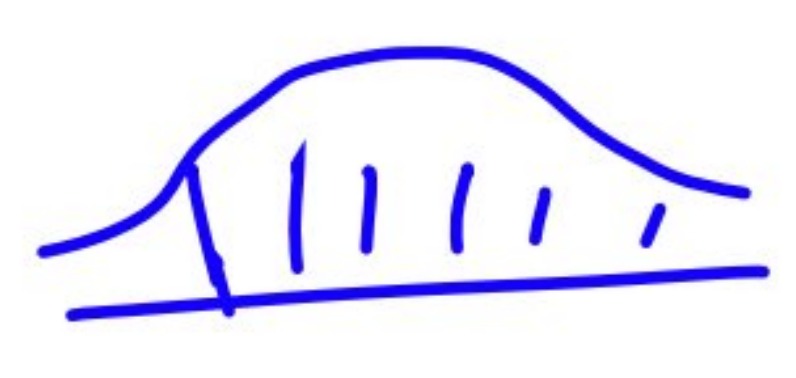
Example

If $X \sim N(\mu, \sigma^2)$, $P(X > 4) = 0.6915$

$P(X \leq 6) = 0.6915$, find μ, σ^2 ?

$$\Rightarrow P(X > 4) = 0.6915$$

$$P\left(Z > \frac{4 - \mu}{\sigma}\right) = 0.6915$$



$$-\frac{1}{2} = \frac{4 - \mu}{\sigma}$$

$$8 - 2\mu = -\sigma$$

$$2\mu - \sigma = 8 \dots \textcircled{1}$$

$$\Rightarrow P(X \leq 6) = 0.6915$$

$$= P\left(Z \leq \frac{6 - \mu}{\sigma}\right) = 0.6915$$



$$\frac{6 - \mu}{\sigma} = \frac{1}{2} \Rightarrow \sigma = 12 - 2\mu$$

$$2\mu + \sigma = 12 \dots \textcircled{2}$$

$$\begin{aligned} -2\mu - \sigma &= 8 \\ -2\mu + \sigma &= 12 \end{aligned}$$

$$-2\sigma = -4$$

$$\sigma = 2$$

$$2\mu - 2 = 8$$

$$2\mu = 10$$

$$\mu = 5$$

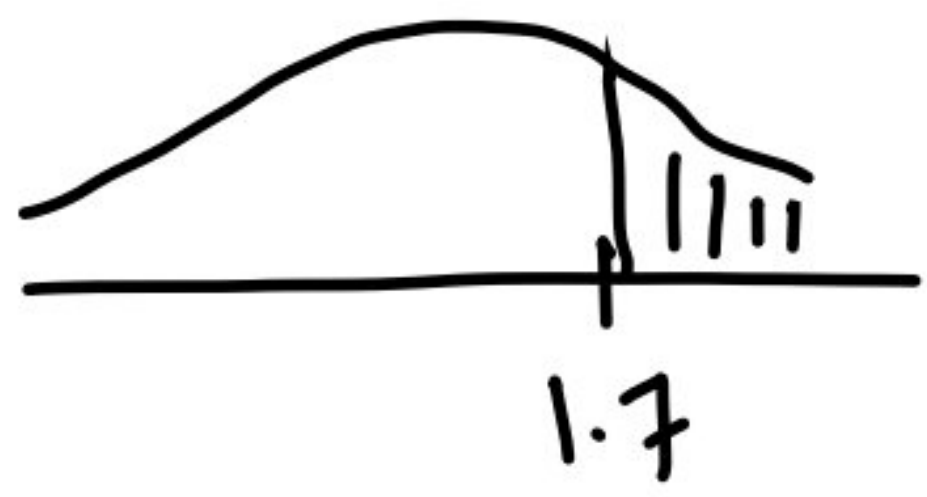
$$\begin{aligned} \sigma^2 &= 4 \\ \mu &= 5 \end{aligned}$$

Example Suppose that the grades in general exam are normally distributed with mean 68 and SD equal to 10, find:

$$X \sim N(68, 10^2)$$

a) the proportion of students that achieved more than 85

$$P(X > 85) = P\left(Z > \frac{85 - 68}{10}\right)$$



$$= P(Z > 1.7)$$

$$= 0.0446$$

B) the proportion of students that achieved between 60 and 90

$$= P(60 < X < 90)$$

$$= P(X < 90) - P(X < 60)$$

$$= P\left(Z < \frac{90 - 68}{10}\right) - P\left(Z < \frac{60 - 68}{10}\right)$$

$$= P(Z < 2.2) - P(Z < -0.8)$$

$$= 0.9861 - 0.2119$$

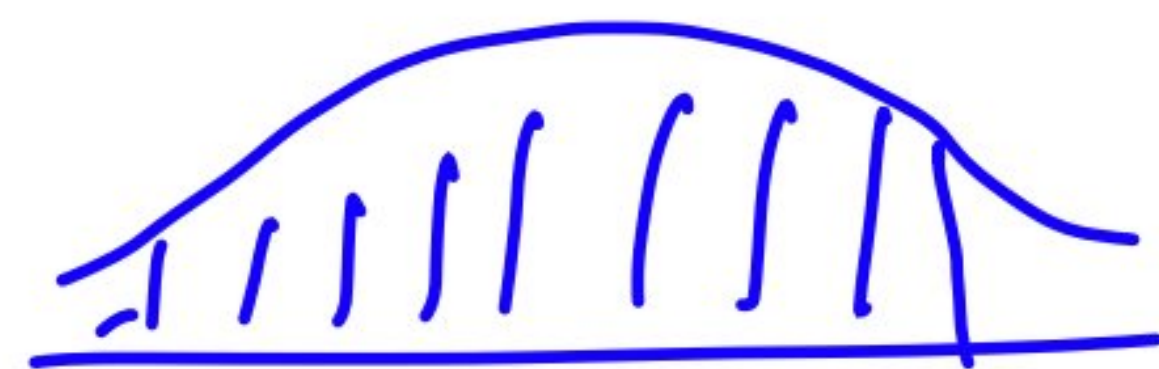
C) 95th percentile (P_{95})

$$P_{95} \Rightarrow P(X \leq P_{95}) = 0.95$$

$$= P\left(Z \leq \frac{P_{95} - 68}{10}\right) = 0.95$$

$$Z = \frac{1.64 + 1.65}{2}$$

$$= 1.645$$



$$1.645 = \frac{P_{95} - 68}{10}$$

Example If heights of students are normally distributed with mean 170 cm and SD 10 cm. Find: $X \sim N(170, 10^2)$

A) a student is selected at random what is the probability that he will be shorter than 170

$$\begin{aligned}
 P(X < 170) &\Rightarrow P\left(Z < \frac{170 - 170}{10}\right) \\
 &= P(Z < 0) \\
 &= 0.5
 \end{aligned}$$

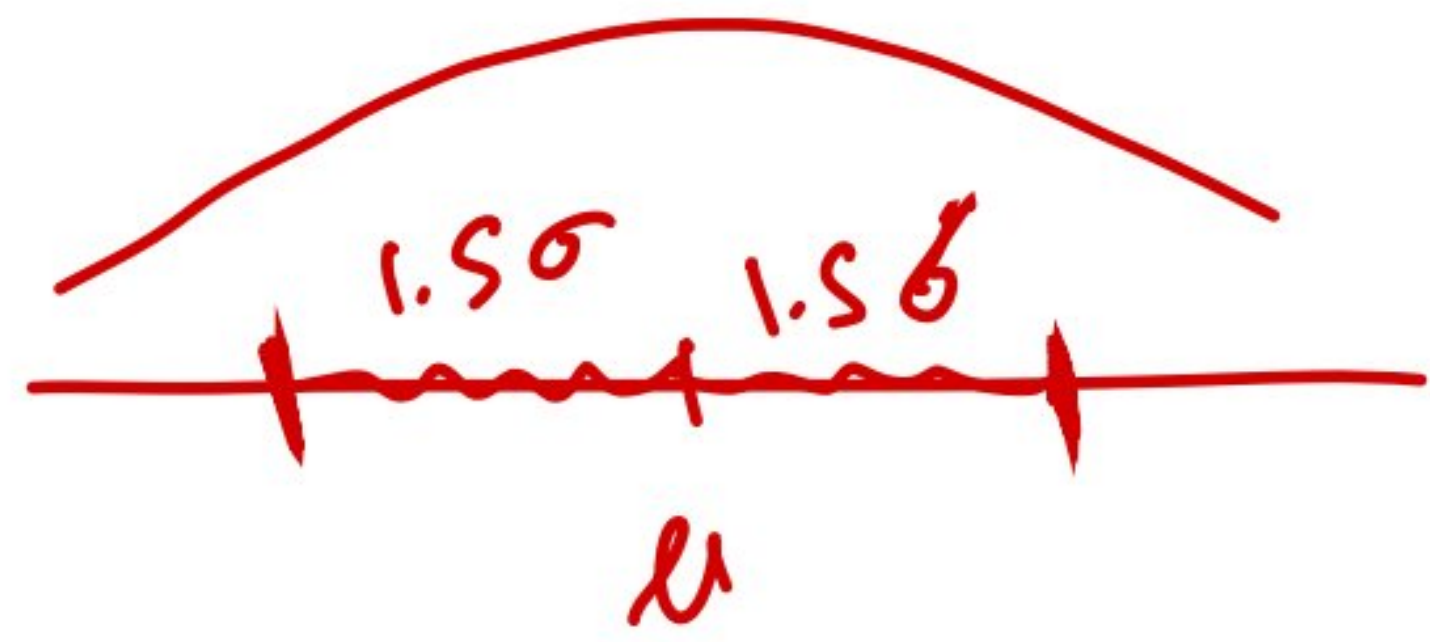
^{مثال}
 B) 10 students are selected at random what is the probability that exactly 4 of them are shorter than 170?

$$X \sim \text{Bin}(10, 0.5)$$

$$P(X=4) = \binom{10}{4} * 0.5^4 * 0.5^6$$

^{مثال}
Example Suppose a child is considered to have normal lung growth if his/her standardized FVC is within 1.5 standard deviation of the mean. what is the proportion of children are within the

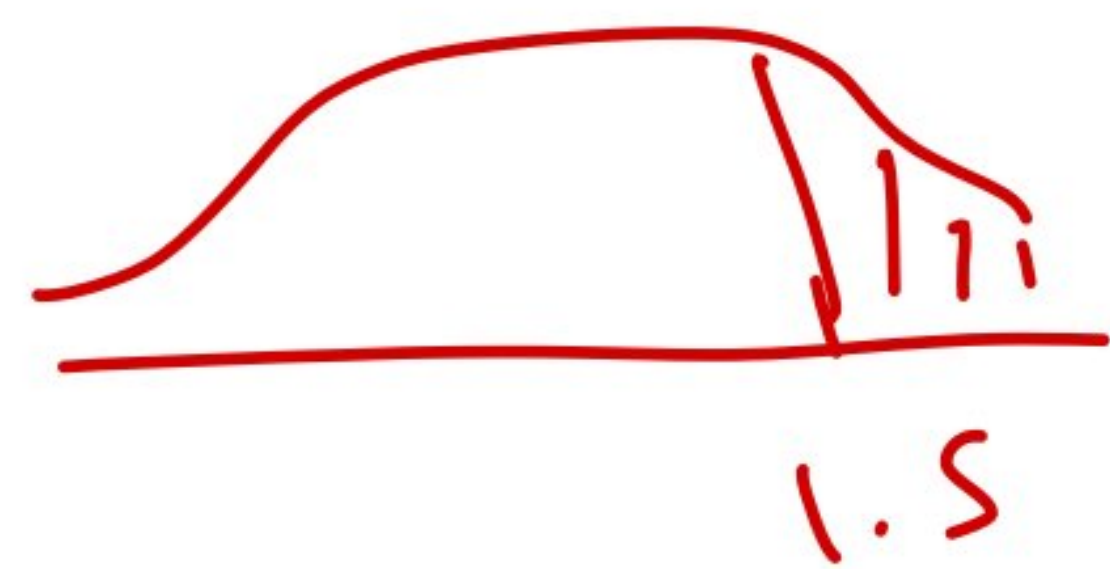
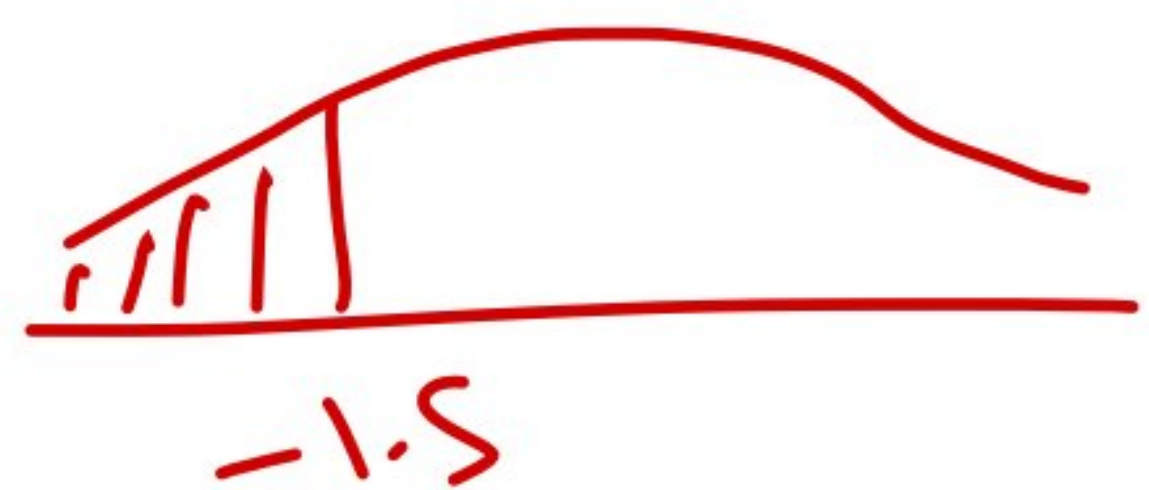
normal range?



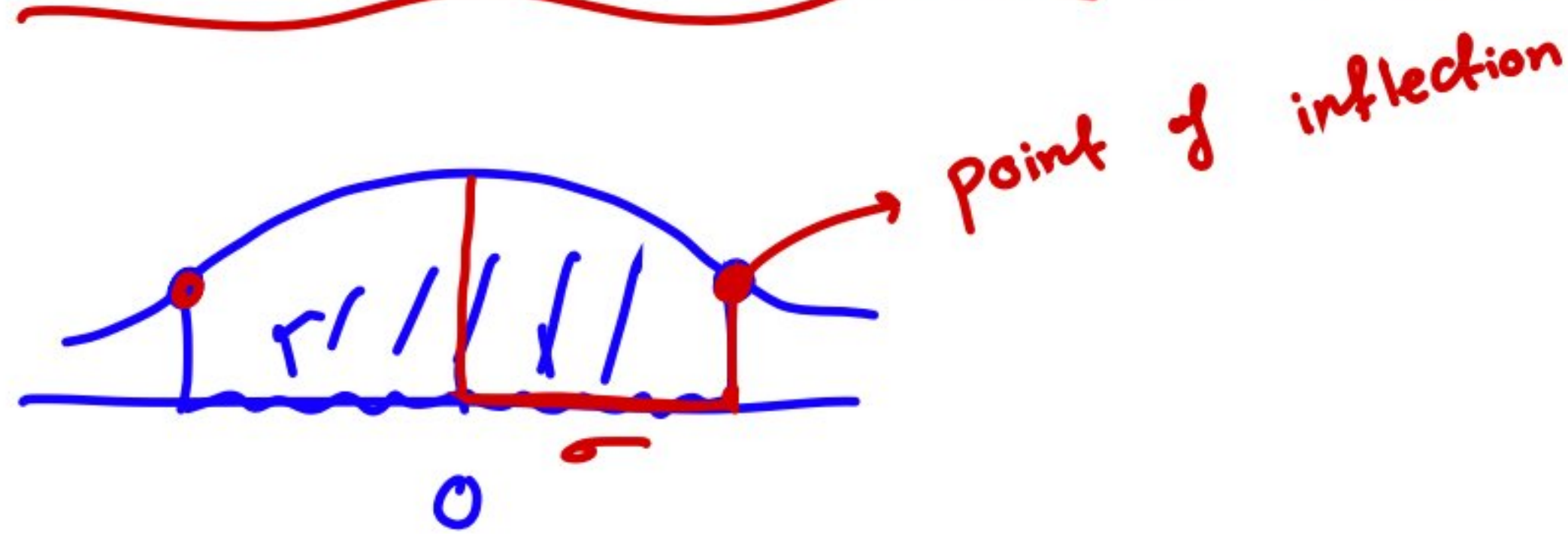
$$\Rightarrow P(-1.5 < Z < 1.5)$$

$$= P(Z < 1.5) - P(Z < -1.5)$$

$$= 0.9332 - 0.0668 = 0.8664$$



NOTE



$$\textcircled{1} P(-1 < Z < 1) = 0.6827$$

$$\textcircled{2} P(-2 < Z < 2) = 0.95$$

$$\textcircled{3} P(-3 < Z < 3) = 0.99$$

NOTE the height of normal distribution curve

is always $f = \frac{1}{\sqrt{2\pi\sigma}}$

$\downarrow \uparrow h \propto \frac{1}{\downarrow \sigma \uparrow}$

Example If $X \sim n(50, 4)$, find:

1) the mean = 50

2) the mode = 50

3) the median = 50

4) IQR = $Q_3 - Q_1 = 51.34 - 48.66 = 2.68$

5) variance $\textcircled{4}$ and SD $\textcircled{2}$

IQR $\Rightarrow Q_3 : P_{75} \Rightarrow P(X \leq P_{75}) = 0.75$
 $= P(Z \leq \frac{P_{75} - 50}{2}) = 0.75$

$$0.67 = \frac{P_{75} - 50}{2}$$

$$P_{75} = 51.34$$

$$Q_1: P_{25} \Rightarrow P(X \leq P_{25}) = 0.25$$

$$\Rightarrow P\left(Z \leq \frac{P_{25} - 50}{2}\right) = 0.25$$

$$-0.67 = \frac{P_{25} - 50}{2}$$

$$P_{25} = 48.66$$